Summer
Homework
Mathematics

ANSWERS

Forming equations

Question 1
- a) $4x + 10$
- b) 12 cm

Question 2
- a) $5x + 120 = 360$ (or $x + 10 + 2x + x + 80 + x + 30 = 360$)
- b) 58°

Question 3
- a) $12x + 18$
- b) $12x + 18 = 48$
- c) Cost of a cup = £2.50; Cost of a mug = £5.50

Rearranging difficult formulae

Question 1

$$c = v - 2a - 3b$$

Question 2

$$t = \frac{A}{\pi + 5}$$

Question 3

$$s = \frac{R - 2t}{3 + \pi}$$

Question 4
- a)

$$l = \frac{km}{1 + k}$$

- b)

$$m = \frac{l(1 + k)}{k} \text{ or } m = \frac{l}{k} + l$$

Question 5

$$x = \frac{3A}{k} - 5 \text{ or } x = \frac{3A - 5k}{k}$$

Question 6

$$u = \frac{v^2 - Rv}{R - 1}$$

Question 7

$$y = \frac{30x + 20}{3 - 3x} \text{ or } y = \frac{10(3x + 2)}{3(1 - x)}$$

Question 8

$$a = 80b^2 + 3$$

Question 9

$$h = \sqrt{\left(\frac{S}{2\pi d}\right)^2 - d^2}$$

Factorisation

Question 1
- a) $2(x + 2)$
- b) $2(y + 5)$
- c) $3(x + 4)$
- d) $3(x - 2)$
- e) $5(x - 3)$

Question 2
- a) $p(p + 7)$
- b) $x(x + 4)$
- c) $y(y - 2)$
- d) $p(p - 5)$
- e) $x(x + 1)$

Question 3
- a) $2x(x + 3)$
- b) $2y(y - 4)$
- c) $5p(p + 2)$
- d) $7c(c - 3)$
- e) $3x(2x + 3)$

Question 4
- a) $2x(x - 2y)$
- b) $2t(t + 5u)$
- c) $2x(3x - 4y)$
- d) $3xy(xy + 3)$

Difference of two squares

Question 1

- a) $(x-4)(x+4)$
- b) $(a-b)(a+b)$
- c) $(y-3)(y+3)$
- d) $(x-1)(x+1)$
- e) $\left(x-\frac{1}{2}\right)\left(x+\frac{1}{2}\right)$
- f) $\left(x-\frac{1}{3}\right)\left(x+\frac{1}{3}\right)$

Question 2

- a) $(x-2y)(x+2y)$
- b) $(3a-b)(3a+b)$
- c) $(3x-4y)(3x+4y)$
- d) $\left(\frac{1}{2}x-y\right)\left(\frac{1}{2}x+y\right)$
- e) $(2x-5y)(2x+5y)$
- f) $\left(x-\frac{1}{3}y\right)\left(x+\frac{1}{3}y\right)$

Question 3

- a) $\frac{5(y-2)}{y+5}$
- b) $\frac{3(2x-1)}{x-2}$
- c) $\frac{4x}{3x-2}$
- d) $\frac{5a+4b}{2b}$

Question 4

- a) $x=2$ or $x=-2$ (or $x=\pm 2$)
- b) $x=\frac{1}{5}$ or $x=-\frac{1}{5}$ (or $x=\pm\frac{1}{5}$)
- c) $x=\frac{11}{7}$ or $x=-\frac{11}{7}$ (or $x=\pm\frac{11}{7}$)
- d) $x=\frac{4}{3}$ or $x=-\frac{4}{3}$ (or $x=\pm\frac{4}{3}$)

Algebraic fractions

Question 1

- a) $\frac{3}{7x}$
- b) $2xy$
- c) $9a^2$
- d) $\frac{2(x+3)}{5}$
- e) $\frac{1-7b}{3ab^2}$
- f) $\frac{x+y}{2xy}$

Question 2

- a) $\frac{x}{x+5}$
- b) $\frac{x-2}{2x}$
- c) $\frac{x}{x+4}$
- d) $\frac{x+2}{x}$

Question 3

- a) $(2x-3)^2$
- b) $\frac{3x+1}{2x-3}$

Question 4

- a) $\frac{9}{2x}$
- b) $\frac{11}{12x}$
- c) $\frac{7x-1}{10}$
- d) $\frac{x-7}{(x+2)(2x+1)}$

Question 5

- a) $(2x+3)(x+2)$
- b) $\frac{10x+9}{(2x+3)(x+2)}$

Question 6

- a) $x=\frac{2}{3}$
- b) $x=6$ or $x=-2$
- c) $x=6$ or $x=\frac{5}{2}$
- d) $x=\frac{3}{2}$ or $x=-\frac{1}{2}$
- e) $x=\frac{11}{4}$
- f) $x=0$ or $x=3$

Drawing quadratics

Question 1

- a) Missing values for the table from left to right:
  - For $x = -1$: 5
  - For $x = 1$: -1
  - For $x = 2$: 2
- b) *Graph plotting coordinates:* $(-2, 14), (-1, 5), (0, 0), (1, -1), (2, 2), (3, 9)$ connected with a smooth parabolic curve.
- c) $y = 9$ (or within the range 8.8 to 9.2)
- d) $x = -0.85$ and $x = 2.35$ (or values near -0.8 to -0.9 and 2.3 to 2.4)

Question 2

- a) Missing values for the table from left to right:
  - For $x = -1$: 3
  - For $x = 1$: -1
  - For $x = 2$: 0
  - For $x = 3$: 3
- b) *Graph plotting coordinates:* $(-2, 8), (-1, 3), (0, 0), (1, -1), (2, 0), (3, 3)$ connected with a smooth parabolic curve.
- c) * (i) *Graph plotting:* A horizontal straight line passing through 2.5 on the y-axis.
  - (ii) $x = -0.9$ and $x = 2.9$ (or values near -0.8 to -1.0 and 2.8 to 3.0)

Solving quadratic equations by factorising

Question 1

- a) $(x+2)(x+3) = 0 \Rightarrow x = -2$ or $x = -3$
- b) $(x+4)(x+5) = 0 \Rightarrow x = -4$ or $x = -5$
- c) $(x+3)(x-2) = 0 \Rightarrow x = -3$ or $x = 2$
- d) $(x+8)(x-3) = 0 \Rightarrow x = -8$ or $x = 3$
- e) $(x-2)(x-4) = 0 \Rightarrow x = 2$ or $x = 4$
- f) $(x-7)(x+4) = 0 \Rightarrow x = 7$ or $x = -4$
- g) $(2x+1)(x+3) = 0 \Rightarrow x = -\frac{1}{2}$ or $x = -3$
- h) $(2x+3)(3x+1) = 0 \Rightarrow x = -\frac{3}{2}$ or $x = -\frac{1}{3}$
- i) $(3x-2)(x+5) = 0 \Rightarrow x = \frac{2}{3}$ or $x = -5$
- j) $(3x-7)(x-9) = 0 \Rightarrow x = \frac{7}{3}$ or $x = 9$

Question 2

- Yes, Lucy is correct. $(x+1)^2 = 0 \Rightarrow x = -1$ is the only solution.

Question 3

- Yes, Ben is correct. $(x+5)^2 = 0 \Rightarrow x = -5$ is the only solution.

Solve quadratic using the quadratic formula

Question 1

- $x = -0.268$ or $x = -3.732$

Question 2

- $x = -0.838$ or $x = -7.16$

Question 3

- $x = 3.56$ or $x = -0.562$

Question 4

- $x = 6.70$ or $x = 0.298$

Question 5

- $x = 0.158$ or $x = -3.16$

Question 6

- $x = 2.92$ or $x = -2.26$

Question 7

- $x = 21.5$ or $x = -7.5$

Question 8

- $x = 7.14$ or $x = -1.70$

Question 9

- $x = 13.2$

Question 10

- a) $(x+2)(x-3) = 1 \Rightarrow x^2 - 3x + 2x - 6 = 1 \Rightarrow x^2 - x - 7 = 0$
- b) $x = 3.19$ or $x = -2.19$

Completing the square

Question 1

- $y=(x+4)^2-19$. Since $(x+4)^2\geq 0$, then $y\geq -19$.

Question 2

- $y=(x-5)^2+5$. Since $(x-5)^2\geq 0$, then $y\geq 5$.

Question 3

- $p=2$
- $q=6$

Question 4

- $p=3$
- $q=8$

Question 5

- a) $p=3, q=-9$
- b) Minimum value = -9

Question 6

- a) $p=4, q=-21$
- b) *Graph sketch:* A standard U-shaped parabola with a y-intercept at $(0,-5)$ and it vertex (lowest point) located in the fourth quadrant at $(4,-21)$.
- c) $(4,-21)$

Question 7

- a) $p=25, q=5$
- b) * (i) Maximum value = 25
  - (ii) $x=5$

Simultaneous equation with a quadratic

Question 1

- $x=3, y=3$ or $x=-2, y=-2$

Question 2

- $x=4, y=12$ or $x=-1, y=-3$

Question 3

- $x=5, y=7$ or $x=-3, y=-1$

Question 4

- $x=6, y=1$ or $x=-5, y=-10$

Question 5

- $x=5, y=-1$ or $x=1, y=-5$

Question 6

- a) Substituting $y=7$ gives $x^2+7^2=25 \Rightarrow x^2+49=25 \Rightarrow x^2=-24$. Since a real number squared cannot be negative, there are no real solutions, showing Sarah is not correct.
- b) $x=4, y=3$ or $x=1.4, y=-4.8$ (or $x=\frac{7}{5}, y=-\frac{24}{5}$)

Quadratic inequalities

Question 1

$-4<x<-2$

Question 2

$x<-7$ or $x>5$

Question 3

$2\leq x\leq 7$

Question 4

$x\leq -5$ or $x\geq 6$

Question 5

$x<-8$ or $x>4$

Question 6

$x<-10$ or $x>-2$

Question 7

$x<-2.5$ or $x>-2$

Question 8

$\frac{8}{7}\leq x\leq 2$

Question 9

$-4<x\leq 0.5$

Question 10

$x<-15$ or $x>10$

Proof

Question 1

$$wy + 4 = (x-2)(x+2) + 4 = x^2 - 4 + 4 = x^2$$

Question 2

$$\frac{6c^3 + 30c}{3c^2 + 15} = \frac{2c(3c^2 + 15)}{3c^2 + 15} = 2c$$

(Since $2c$ has a factor of 2, it is always an even number)

Question 3

$$\text{LHS} = \frac{(3n+5)(n-1) - 3n(n)}{3n(n-1)} = \frac{(3n^2 + 2n - 5) - 3n^2}{3n(n-1)} = \frac{2n-5}{3n(n-1)}$$

Question 4

$$x^2 + x + 1 = \left(x + \frac{1}{2}\right)^2 + \frac{3}{4}$$

(Since $\left(x+\frac{1}{2}\right)^2 \geq 0$, the expression is always $\geq \frac{3}{4}$, which is always positive)

Question 5

$$5n - (2n+3)(n+1) = 5n - (2n^2 + 5n + 3) = -2n^2 - 3$$

(Since $2n^2 \geq 0$, then $-2n^2 - 3 \leq -3$, which is always negative)

Question 6

$$n^2 + 3n + 2 = (n+1)(n+2)$$

(As a product of two consecutive integers, one must be even, making the product a multiple of 2)

Question 9

$$(2n+1)^2 = 4n^2 + 4n + 1 = 4n(n+1) + 1$$

(Since either n or $n+1$ is even, $n(n+1)$ is a multiple of 2, meaning $4n(n+1)$ is a multiple of 8, leaving a remainder of 1)

Trigonometry

Question 1

- $x = 22.2°$

Question 2

- $\text{BC} = 11.6 \text{ cm}$

Question 3

- $x = 27.8°$

Question 4

- $\text{AC} = 64.2 \text{ cm}$

Question 5

- $x = 27.9°$

Bearing by trig

Question 1

- a) $037.8°$ (or $37.8°$)
- b) $217.8°$

Question 2

- 19.1 km

Sine and cosine rules

Question 1

- $x = 36.2°$

Question 2

- $\text{AB} = 6.27 \text{ cm}$

Question 3

- Largest angle = $111.9°$

Question 4

- $x = 13.9 \text{ cm}$
- $y = 11.2 \text{ cm}$

Area of triangles

Question 1

- $\text{Area} = 26.8 \text{ cm}^2$

Question 2

- $\text{Area} = 106 \text{ cm}^2$

Question 3

- $\text{Area} = 357 \text{ cm}^2$

3D coordinates

Question 1

- a) $\text{P} = (5, 3, 0)$
- b) $\text{T} = (5, 0, 0)$
- c) $\text{S} = (5, 0, 4)$
- d) $\text{R} = (0, 0, 4)$
- e) $\text{U} = (0, 3, 0)$

Question 2

- a) $\text{B} = (6, 4, 5)$
- b) $\text{C} = (6, 4, 0)$

Pythagoras in 3D

Question 1

- $\text{AG} = 7.81 \text{ cm}$

Question 2

- $\text{AE} = 11.2 \text{ cm}$

Question 3

- $\text{AC} = 27.5 \text{ cm}$

Question 4

- $\text{PQ} = 5.9$

Trig in 3D

Question 1

- Angle = 17.1°

Question 2

- a) $\text{CE} = 55.9 \text{ cm}$
- b) Angle = 26.6°

Graph of trig functions 1

Question 1

- **Sketch:** A standard sine wave curve starting at $(0, 0)$, peaking at $(90, 1)$, crossing the x-axis at $(180, 0)$, reaching a minimum at $(270, -1)$, and ending at $(360, 0)$.
- **(i)** 0.5
- **(ii)** -0.5

Question 2

- **Sketch:** A standard cosine wave curve starting at $(0, 1)$, crossing the x-axis at $(90, 0)$, reaching a minimum at $(180, -1)$, crossing the x-axis at $(270, 0)$, and ending at $(360, 1)$.
- **(i)** -0.5
- **(ii)** -0.5

Graph of trig functions 2

Question 1

- **Sketch:** A standard repeating tangent graph with vertical asymptotes at $x = 90^\circ$ and $x = 270^\circ$. The curve starts at $(0, 0)$, goes up to infinity towards $x = 90^\circ$, then comes from negative infinity through $(180, 0)$ up to positive infinity towards $x = 270^\circ$, and finally comes up from negative infinity to meet $(360, 0)$.

Question 2

- **a)** $x = 41.4^\circ$ and $x = 318.6^\circ$ (or values near 41° and 319°)
- **b)** $x = 138.6^\circ$ and $x = 221.4^\circ$ (or values near 139° and 221°)

Transformations of functions

Question 1

- a) (i) $(4, -1)$
  - (ii) $(1, -6)$
  - (iii) $(1, 1)$
  - (iv) $(0.5, -1)$
- b) $f(x) = (x-1)^2 - 1$ (or $f(x) = x^2 - 2x$)

Question 2

- a) *Sketch description:* The entire curve shifts 1 unit to the right. The local minimum moves from $(-1, 0)$ to $(0, 0)$, and the local maximum moves from $(0, 1)$ to $(1, 1)$.
- b) *Sketch description:* The curve stretches vertically by a factor of 2. The local minimum remains at $(-1, 0)$, but the local maximum stretches upward from $(0, 1)$ to $(0, 2)$.

Question 3

- **Sketch description:** A standard parabola shifted right 2 units and up 3 units.
- **Vertex:** $(2, 3)$

Question 4

- **Vertex:** $(-2, -5)$
- **x-intercepts:** $(-2+\sqrt{5}, 0)$ and $(-2-\sqrt{5}, 0)$ (or approximately $(0.24, 0)$ and $(-4.24, 0)$)

Transformations of trig functions

Question 1

- a) (i) $\mathrm{A} = (180, 0)$
  - (ii) $\mathrm{B} = (90, 2)$
- b) *Sketch description:* The entire curve shifts vertically upwards by 1 unit. The curve now oscillates between $y = 3$ and $y = -1$, starting at $(0, 1)$, reaching its maximum point at $(90, 3)$, crossing the line $y = 1$ at $x = 180°$, reaching its minimum point at $(270, -1)$, and ending at $(360, 1)$.

Question 2

- $a = 2$
- $b = 3$

Iterations

Question 1

- a) Let $f(x) = x^3 + 2x - 1$.
  $f(0) = -1$ (negative) and $f(1) = 2$ (positive). Since there is a change of sign, a solution lies between $x = 0$ and $x = 1$.
- b) $x^3 + 2x = 1 \Rightarrow 2x = 1 - x^3 \Rightarrow x = \frac{1}{2} - \frac{x^3}{2}$
- c) $x_1 = 0.5, x_2 = 0.4375$

Question 2

- a) Let $f(x) = 3x - x^3 + 11$.
  $f(2) = 9$ (positive) and $f(3) = -7$ (negative). Since there is a change of sign, a solution lies between $x = 2$ and $x = 3$.
- b) $3x - x^3 = -11 \Rightarrow x^3 = 3x + 11 \Rightarrow x = \sqrt[3]{3x+11}$
- c) $x_1 = 2.714, x_2 = 2.675, x_3 = 2.669$ (values rounded to 3 decimal places)

Question 3

- a) $20 - x^3 - 7x^2 = 0 \Rightarrow x^3 + 7x^2 = 20 \Rightarrow x^2(x+7) = 20 \Rightarrow x + 7 = \frac{20}{x^2} \Rightarrow x = \frac{20}{x^2} - 7$
- b) $x_1 = -6.753, x_2 = -6.561, x_3 = -6.536$ (values rounded to 3 decimal places)
- c) They represent successive approximations that converge toward a root of the equation $20 - x^3 - 7x^2 = 0$.

Direct and inverse proportion

Question 1

- a) $x = 7y$
- b) (i) $x = 7$
  - (ii) $x = 14$
  - (iii) $x = 70$

Question 2

- a) $a = \frac{48}{b}$
- b) (i) $a = 48$
  - (ii) $a = 6$
  - (iii) $a = 4.8$
- c) (i) $b = 12$
  - (ii) $b = 2$
  - (iii) $b = 15$

Question 3

- $u = 2$

Question 4

- a) $p = 3q^2$
- b) $p = 147$
- c) $q = 3$

Question 5

- a) $y = 4x^2$
- b) $z = 16y^{-0.5}$ (where $c = 16$ and $n = -0.5$)

Surds

Question 1

- a) 7
- b) 3
- c) $2\sqrt{5}$
- d) $2\sqrt{6}$
- e) $6\sqrt{2}$
- f) $10\sqrt{2}$
- g) $\frac{\sqrt{2}}{5}$

Question 2

- a) 6
- b) 16
- c) $33\sqrt{2}$
- d) 30
- e) 48
- f) 70

Question 3

- a) $3\sqrt{3}-3$
- b) $6\sqrt{2}+4$
- c) $2\sqrt{7}+21$
- d) 4

Question 4

- a) -1
- b) $1-\sqrt{5}$
- c) $11+6\sqrt{3}$
- d) $2-2\sqrt{5}$
- e) -3
- f) $15-6\sqrt{6}$

Question 5

- a) $\frac{3\sqrt{2}}{2}$
- b) $\sqrt{2}$
- c) $\frac{3\sqrt{14}}{7}$
- d) $\frac{\sqrt{2}}{2}$
- e) $\frac{\sqrt{2}}{8}$
- f) $\sqrt{5}$
- g) $\frac{\sqrt{3}}{9}$

Question 6

- $n = 2.5$ (or $\frac{5}{2}$)

Question 7

- $16\sqrt{2}\,(m = 16)$

Question 8

- $\frac{\sqrt{2}}{32}\,(p = 32)$

Question 9

- a) $\sqrt{22}$
- b) $\sqrt{11}$
- c) $\frac{\sqrt{14}}{2}$
- d) $\frac{4\sqrt{3}}{3}+2$
- e) $\frac{7\sqrt{5}}{5}+3$
- f) $\sqrt{5}$

Fractional and negative indices

Question 1

- a) p^{25}
- b) k^5
- c) x^3
- d) p^{-6} (or $\frac{1}{p^6}$)
- e) m^{10}
- f) $27x^3y^6$

Question 2

- a) 16
- b) 25
- c) 49
- d) 6
- e) 1
- f) 64

Question 3

- a) 1
- b) 1
- c) 1
- d) 1
- e) $\frac{1}{16}$
- f) $\frac{1}{8}$
- g) $\frac{1}{125}$
- h) $\frac{1}{100000}$
- i) 7
- j) 4
- k) 3
- l) 64
- m) $\frac{1}{7}$
- n) $\frac{1}{4}$
- o) $\frac{1}{3}$
- p) $\frac{1}{64}$

Question 4

- $n = 1.5$ (or $\frac{3}{2}$)

Question 5

- $m = 2.5$ (or $\frac{5}{2}$)

Question 6

- $x = 1.5$ (or $\frac{3}{2}$)

Question 7

- $y = 3.5$ (or $\frac{7}{2}$)

Question 8

- a) (i) ab
  - (ii) a^2
  - (iii) a^2b^2
- b) $x = 3, y = 1$

Equations of circle and loci

Question 1

- **Proof outline:** The equation $x^2 + y^2 = 9$ represents a circle centered at the origin $(0, 0)$ with a radius of 3.
  The distance from the origin $(0, 0)$ to the given point $(1, 1)$ is calculated using the distance formula:

$$\text{Distance} = \sqrt{(1-0)^2 + (1-0)^2} = \sqrt{2} \approx 1.41$$

Since the distance (≈ 1.41) is strictly less than the radius of the circle (3), the point $(1, 1)$ lies entirely inside the circle. Any straight line passing through an interior point of a circle must cross its perimeter at exactly two distinct points.

Question 2

- **Proof outline:** The distance from a general point $P(x, y)$ to the point $(0, 3)$ is:

$$\text{Distance}_1 = \sqrt{(x-0)^2 + (y-3)^2} = \sqrt{x^2 + (y-3)^2}$$

The perpendicular distance from the point $P(x, y)$ to the x-axis is simply its y-coordinate:

$$\text{Distance}_2 = y$$

Equating the two distances based on the given property:

$$\sqrt{x^2 + (y-3)^2} = y$$

Squaring both sides removes the square root:

$$x^2 + (y-3)^2 = y^2$$

$$x^2 + y^2 - 6y + 9 = y^2$$

Subtracting y^2 from both sides gives:

$$x^2 - 6y + 9 = 0$$

$$6y = x^2 + 9$$

$$y = \frac{x^2 + 9}{6}$$

Cubic and reciprocal graphs

Question 1

- **a) Completed Table:**

x	-2	-1	0
y	-14	-6	-4

...

- **b) Graph:** Plot the points $(-2,-14)$, $(-1,-6)$, $(0,-4)$, $(1,-2)$, and $(2,6)$ on the grid and connect them with a smooth, continuous cubic curve.
- **c)** $x \approx 1.6$ (or the value read directly from your plotted curve where it crosses the line $y = 2$)

Question 2

- **a) Completed Table:**

x	-2	-1	0
y	-12	-3	0

...

- **b) Graph:** Plot the points $(-2,-12)$, $(-1,-3)$, $(0,0)$, $(1,3)$, and $(2,12)$ on the grid and connect them with a smooth, continuous cubic curve passing through the origin.
- **c)** $x \approx -1.5$ (or the value read directly from your plotted curve where it crosses the line $y = -6$)

Question 3

- **Sketch description:** A standard reciprocal graph ($y = \frac{1}{x}$) shifted vertically upwards by 1 unit. It consists of two curves (hyperbolas) located in the upper-right and lower-left sections relative to its horizontal asymptote at $y = 1$ and a vertical asymptote along the y-axis ($x = 0$). The right curve passes through $(1, 2)$ and approaches $y = 1$, while the left curve passes through $(-1, 0)$ and crosses the x-axis there.

Recognise the shape of a function

Row 1:

- **Left:** $y = 2^x$
- **Right:** $y = 2x^2 + 1$

Row 2:

- **Left:** $y = 5x - x^3$
- **Right:** $y = \frac{2}{x}$

Row 3:

- **Left:** $y = 3x^3$
- **Right:** $y = -2x^3$

Row 4:

- **Left:** $y = \frac{-2}{x}$
- **Right:** $y = 3x - 1$

Graphs of exponential functions

1)

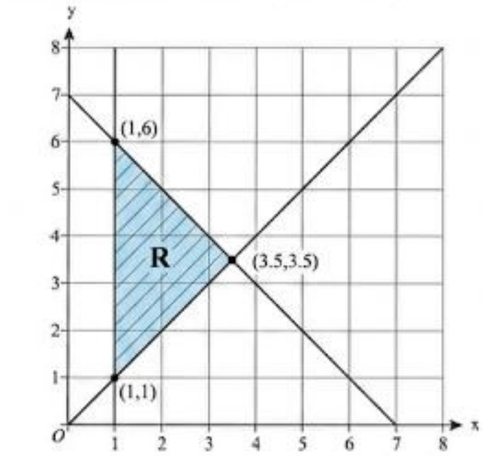
- $p = 0.6$ (or $\frac{3}{5}$)
- $q = 5$

2)

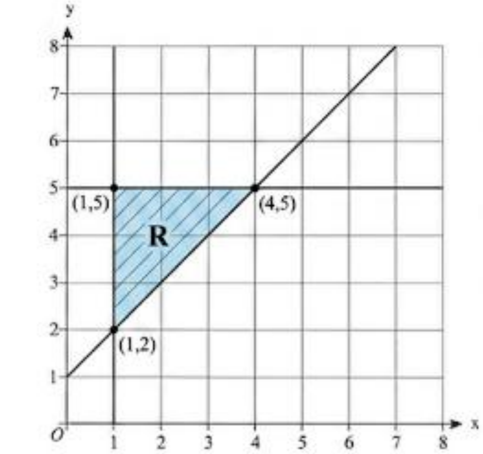
- **a)** $400 = a \times b^0 \Rightarrow 400 = a \times 1 \Rightarrow a = 400$
- **b)** $b = 1.5$
- **c)** 1350

Regions

1) On the grid below, draw straight lines and use shading to show the region **R** that satisfies the inequalities $x > 1$ $\quad y > x \quad$ $x + y < 7$



2) On the grid below, draw straight lines and use shading to show the region **R** that satisfies the inequalities $y > x + 1 \quad y \leqslant 5 \quad x > 1$



Vectors - 1

Question 1

Proof:

- $XY = XO + OA + AY = -2\mathbf{b} + \mathbf{a} + \frac{1}{2}(3\mathbf{b} - \mathbf{a}) = \frac{1}{2}(\mathbf{a} - \mathbf{b})$
- $XZ = XO + OZ = -2\mathbf{b} + 2\mathbf{a} = 2(\mathbf{a} - \mathbf{b})$
- Since $XZ = 4XY$, the vectors are parallel. Because they share the common point X, the points X, Y, and Z lie on a straight line.

Question 2

$k = \frac{1}{2}$

Question 3

$k = \frac{1}{4}$

Question 4

$k = 4$

Vectors – 2

Question 1

a) $\mathbf{p} + \mathbf{q}$

b) Proof:

- $SR = PR - PS = (\mathbf{p} + \mathbf{q}) - 2\mathbf{q} = \mathbf{p} - \mathbf{q}$
- $RX = PR = \mathbf{p} + \mathbf{q}$
- $SX = SR + RX = (\mathbf{p} - \mathbf{q}) + (\mathbf{p} + \mathbf{q}) = 2\mathbf{p}$
- Since $SX = 2PQ$, the vectors are scalar multiples of each other, proving that PQ is parallel to SX.

Question 2

a) $3\mathbf{a} - 3\mathbf{b}$

b) Proof:

- $XY = XM + MC + CY$
- $XM = -\frac{1}{2}MN = -\frac{1}{2}(3\mathbf{a} - 3\mathbf{b}) = -1.5\mathbf{a} + 1.5\mathbf{b}$
- $MC = \frac{1}{2}BC = 1.5\mathbf{a}$
- $CD = CB + BA + AD = -3\mathbf{a} - 3\mathbf{b} + 9\mathbf{a} = 6\mathbf{a} - 3\mathbf{b}$
- $CY = \frac{1}{2}CD = 3\mathbf{a} - 1.5\mathbf{b}$
- $XY = (-1.5\mathbf{a} + 1.5\mathbf{b}) + 1.5\mathbf{a} + (3\mathbf{a} - 1.5\mathbf{b}) = 3\mathbf{a}$
- Since $XY = 3\mathbf{a}$ and $AD = 9\mathbf{a}$, $AD = 3XY$. Being scalar multiples, XY is parallel to AD.

Averages from a table

Question 1

a) 52
b) 2
c) 2
d) 5

Question 2

a) Thomas has confused the highest frequency with the mode. The highest frequency is 7, which corresponds to 0 goals scored. Therefore, the true mode is 0.
b) 1.92

Question 3

a) $30 < t \leq 45$
b) 37 minutes

Scatter diagrams

Question 1

a) 52
b) 2
c) 2
d) 5

Question 2

a) Thomas has confused the highest frequency with the mode. The highest frequency is 7, which corresponds to 0 goals scored. Therefore, the true mode is 0.
b) 1.92

Question 3

a) $30 < t \leq 45$
b) 37 minutes

Cumulative Frequency

c) * (i) Median: ~33.7 cm

- (ii) Interquartile Range: ~12.2 cm

d) 9 or 10 plants

Box plots

Question 1: Teachers' Ages

- **Minimum:** 22
- **Lower Quartile (Q1):** 27 *(average of the 5th and 6th values: (27 + 27) / 2)*
- **Median (Q2):** 31.5 *(average of the 10th and 11th values: (29 + 34) / 2)*
- **Upper Quartile (Q3):** 46.5 *(average of the 15th and 16th values: (44 + 49) / 2)*
- **Maximum:** 58

Question 2: Warehouse Employees' Ages

- **Minimum:** 16
- **Lower Quartile (Q1):** 29
- **Median (Q2):** 37
- **Upper Quartile (Q3):** 43
- **Maximum:** 55

Histograms

Question 1: Frequency Densities

To draw a histogram, you need to calculate the **Frequency Density** for each class interval. The formula is:

$\text{Frequency Density} = \text{Frequency} \div \text{Class Width}$

Here are the calculations for each row:

Height (h cm)	Frequency	Class Width	Frequency De
$135 < h \leq 145$	12	10	**1.2** ($12 \div 10$)
$145 < h \leq 165$	46	20	**2.3** ($46 \div 20$)
$165 < h \leq 180$	45	15	**3.0** ($45 \div 15$)
$180 < h \leq 190$	25	10	**2.5** ($25 \div 10$)
$190 < h \leq 195$	4	5	**0.8** ($4 \div 5$)

Question 2: Completing the Table

In a histogram, the area of the bar is proportional to the frequency ($\text{Frequency} = \text{Frequency Density} \times \text{Class Width}$).

First, let's find the scale using the given value: The bar for $2 < t \leq 3$ has a frequency of 27. It is 1 unit wide on the x-axis and 27 small squares high on the y-axis. This means 1 small square on the y-axis represents 1 unit of Frequency Density.

Now we can calculate the rest of the frequencies:

- $0 < t \leq \frac{1}{2}$: Width is 0.5. Height is 15. Frequency = $15 \times 0.5 =$ **7.5**
- $\frac{1}{2} < t \leq 1$: Width is 0.5. Height is 24. Frequency = $24 \times 0.5 =$ **12**
- $1 < t \leq 2$: Width is 1. Height is 36. Frequency = $36 \times 1 =$ **36**
- $2 < t \leq 3$: (Given as **27**)
- $3 < t \leq 5$: Width is 2. Height is 6. Frequency = $6 \times 2 =$ **12**

Completed Table:

Time (t hours)	Frequency
$0 < t \leq \frac{1}{2}$	**7.5**
$\frac{1}{2} < t \leq 1$	**12**
$1 < t \leq 2$	**36**
$2 < t \leq 3$	**27**
$3 < t \leq 5$	**12**

Venn diagrams

Question 1

- **b)** 1/16

Question 2

- 4

Question 3

- 29%

Tree diagrams

Question 1

- **b)** 42/100 (or 21/50)

Question 2

- **b)** 42/90 (or 7/15)

Question 3

- **b)** 36/56 (or 9/14)
- **c)** 26/56 (or 13/28)
- **d)** 30/56 (or 15/28)

Question 4

- **b)** 30/210 (or 1/7)
- **c)** 108/210 (or 18/35)

And & or questions

Question 1

- 18/64 (or 9/32)

Question 2

- **a)** 0.0036
- **b)** 0.1128

Question 3

- 660/1000 (or 33/50 or 0.66)