Summer Homework Mathematics

This booklet has been designed to prepare you for the Sixth Form at St. Michael's Catholic Grammar School.

In order to access the challenges of A Level Mathematic, it is important to be prepared. We require you to have a number of basic key skills that you will have learnt during your GCSE course and will be called upon during your time in both Yr 12 and Yr 13 Mathematics. The questions must be completed, and you should bring any concerns to your Maths' teachers in the first week in September (maths clinics are held every Tuesdays from 1.20 pm to 2.00 pm). If you are not sure of any topic, use a GCSE textbook for reference

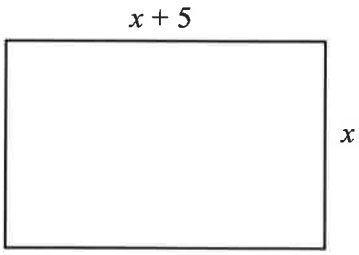
Note: there will be two Key Skills Tests on these topics in September.

Please order now your textbooks as you will need them from the first lesson

- Edexcel AS and A level Mathematics Pure Mathematics Year 1/AS Textbook + e-book - ISBN 9781292183398
- Edexcel AS and A level Mathematics Statistics & Mechanics Year 1/AS Textbook + e-book - ISBN 9781292232539
- Edexcel A level Mathematics Pure Mathematics Year 2 Textbook + e-book - ISBN 9781292183404
- Edexcel A level Mathematics Statistics & Mechanics Year 2 Textbook + e-book - ISBN 9781446944073

Forming Equations

1) The width of a rectangle is x centimetres.
The length of the rectangle is $(x + 5)$ centimetres.



a) Find an expression, in terms of x, for the perimeter of the rectangle.
Give your answer in its simplest form.

The perimeter of the rectangle is 38 centimetres.

b) Work out the length of the rectangle.

2)

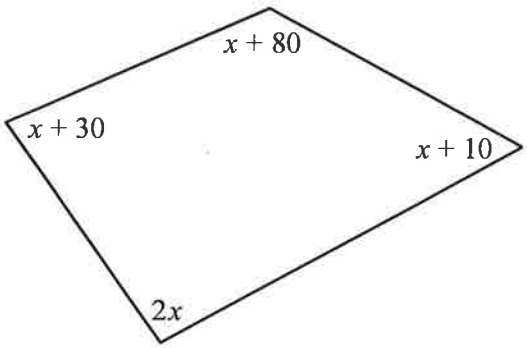


*Diagram **NOT** accurately drawn*

The sizes of the angles, in degrees, of the quadrilateral are

$x + 10$
$2x$
$x + 80$
$x + 30$

a) Use this information to write down an equation in terms of x.

b) Use your answer to part (a) to work out the size of the smallest angle of the quadrilateral.

3) Sarah buys 6 cups and 6 mugs
A cup costs £x
A mug costs £$(x + 3)$

a) Write down an expression, in terms of x, for the total cost, in pounds, of 6 cups and 6 mugs.

b) If the total cost of 6 cups and 6 mugs is £48, write an equation in terms of x.

c) Solve your equation to find the cost of a cup and the cost of a mug.

Rearranging Difficult Fomulae

1) Make c the subject of the formula.

$v = 2a + 3b + c$

2) Make t the subject of the formula.

$A = \pi t + 5t$

3) Make s the subject of the formula.

$R = 3s + \pi s + 2t$

4) $k = \dfrac{l}{m - l}$

a) Make l the subject of the formula.

b) Make m the subject of the formula.

5) $A = \dfrac{k(x+5)}{3}$

Make x the subject of the formula.

6) $R = \dfrac{u + v^2}{u + v}$

Make u the subject of the formula.

7) $\dfrac{3x+2}{5} = \dfrac{y}{10+y}$

Make y the subject of the formula.

8) $\sqrt{\dfrac{a-3}{5}} = 4b$

Rearrange this formula to give a in terms of b.

9) $S = 2\pi d\sqrt{h^2 + d^2}$

Rearrange this formula to make h the subject.

Factorisation

1) Factorise

   a) $2x + 4$

   b) $2y + 10$

   c) $3x + 12$

   d) $3x - 6$

   e) $5x - 15$

2) Factorise

   a) $p^2 + 7p$

   b) $x^2 + 4x$

   c) $y^2 - 2y$

   d) $p^2 - 5p$

   e) $x^2 + x$

3) Factorise

   a) $2x^2 + 6x$

   b) $2y^2 - 8y$

   c) $5p^2 + 10p$

   d) $7c^2 - 21c$

   e) $6x^2 + 9x$

4) Factorise

   a) $2x^2 - 4xy$

   b) $2t^2 + 10tu$

   c) $6x^2 - 8xy$

   d) $3x^2y^2 + 9xy$

Difference of Two Squares

$$x^2 - y^2 = (x - y)(x + y)$$

1) Factorise

a) $x^2 - 16$

b) $a^2 - b^2$

c) $y^2 - 9$

d) $x^2 - 1$

e) $x^2 - \frac{1}{4}$

f) $x^2 - \frac{1}{9}$

2) Factorise

a) $x^2 - 4y^2$

b) $9a^2 - b^2$

c) $9x^2 - 16y^2$

d) $\frac{1}{4}x^2 - y^2$

e) $4x^2 - 25y^2$

f) $x^2 - \frac{1}{9}y^2$

3) Simplify

a) $\frac{y^2-4}{y+2} \times \frac{5}{y+5}$

b) $\frac{3}{2x+1} \times \frac{4x^2-1}{x-2}$

c) $\frac{12x^2+8x}{9x^2-4}$

d) $\frac{25a^2-16b^2}{10ab-8b^2}$

4) Solve

a) $4x^2 - 16 = 0$

b) $25x^2 = 1$

c) $49x^2 = 121$

d) $9x^2 - 9 = 7$

Algebraic Fractions

1) Simplify fully

a) $\frac{9x^2}{21x^3}$

b) $\frac{10xy^3}{5y^2}$

c) $\frac{18a^3b^2}{2ab^2}$

d) $\frac{4x^2+12x}{10x}$

e) $\frac{2a^2b-14a^2b^3}{6a^3b^3}$

f) $\frac{5x^2y+5xy^2}{10x^2y^2}$

2) Simplify fully

a) $\frac{x^2+x}{x^2+6x+5}$

b) $\frac{x^2-6x+8}{2x^2-8x}$

c) $\frac{x^2-3x}{x^2+x-12}$

d) $\frac{x^2+7x+10}{x^2+5x}$

3) a) Factorise $4x^2-12x+9$

b) Simplify $\frac{6x^2-7x-3}{4x^2-12x+9}$

4) Write as single fractions in their simplest form

a) $\frac{3}{x}+\frac{3}{2x}$

b) $\frac{5}{3x}-\frac{3}{4x}$

c) $\frac{x+2}{5}+\frac{x-1}{2}$

d) $\frac{3}{x+2}-\frac{5}{2x+1}$

5) a) Factorise $2x^2+7x+6$

b) Write as a single fraction in its simplest form $\frac{3}{x+2}+\frac{4x}{2x^2+7x+6}$

6) Solve

a) $\frac{1}{x}+\frac{1}{3x}=2$

b) $\frac{1}{x-2}+\frac{3}{x+6}=\frac{1}{2}$

c) $\frac{1}{x-5}+\frac{6}{x}=2$

d) $\frac{7}{x+2}+\frac{1}{x-1}=4$

e) $\frac{3}{x+2}+\frac{1}{x-2}=\frac{7}{x^2-4}$

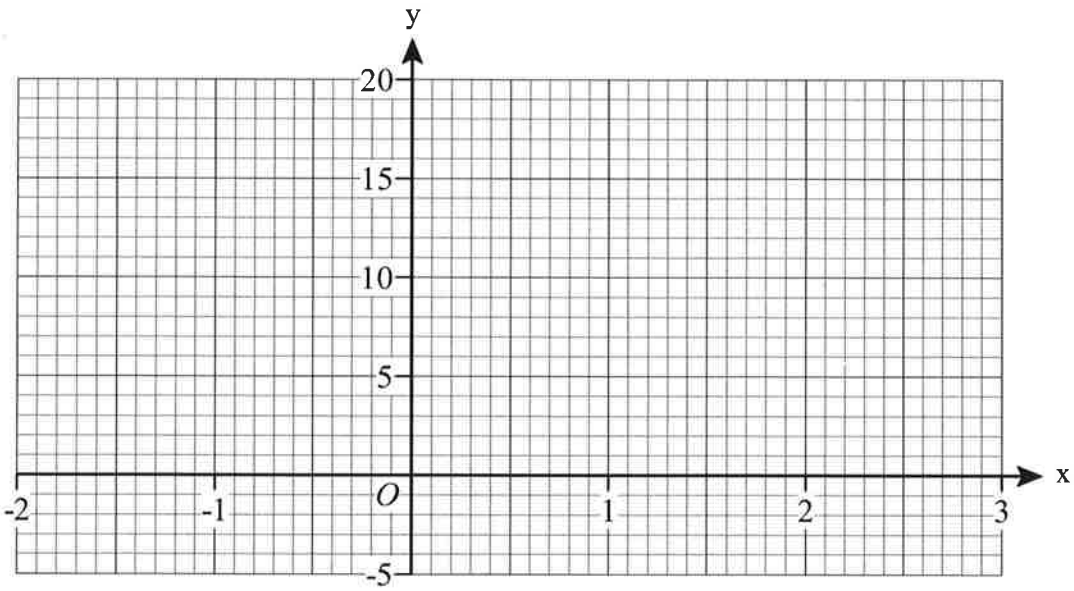
f) $\frac{x}{2x-1}+\frac{2}{x+2}=1$

Drawing Quadratic Graphs

1) a) Complete the table of values for $y = 2x^2 - 3x$

x	-2	-1	0	1	2	3
y	14		0			9

b) On the grid, draw the graph of $y = 2x^2 - 3x$ for values of x from -2 to 3



c) Use the graph to find the value of y when x = -1.5

d) Use the graph to find the values of x when y = 4

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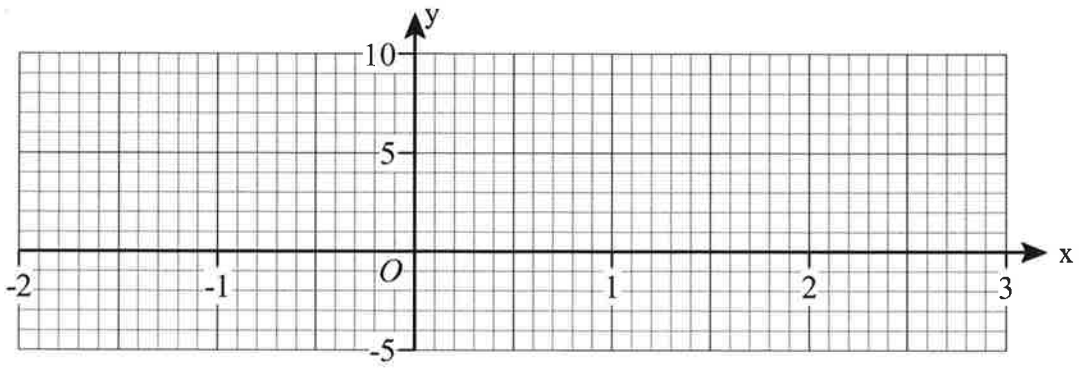
2) a) Complete the table of values for $y = x^2 - 2x$

x	-2	-1	0	1	2	3
y	8		0			

b) On the grid, draw the graph of $y = x^2 - 2x$ for values of x from -2 to 3



c) (i) On the same axes draw the straight line y = 2.5

(ii) Write down the values of x for which $x^2 - 2x = 2.5$

Solving Quadratic Equations by Factorising

1) Factorise and solve the following equations:

a) $x^2 + 5x + 6 = 0$

b) $x^2 + 9x + 20 = 0$

c) $x^2 + x - 6 = 0$

d) $x^2 + 5x - 24 = 0$

e) $x^2 - 6x + 8 = 0$

f) $x^2 - 3x - 28 = 0$

g) $2x^2 + 7x + 3 = 0$

h) $6x^2 + 11x + 3 = 0$

i) $3x^2 + 13x - 10 = 0$

j) $3x^2 - 34x + 63 = 0$

2) Lucy said that -1 is the only solution of x that satisfies the equation $x^2 + 2x + 1 = 0$

Was Lucy correct?
Show working to justify your answer

3) Ben said that -5 is the only solution of x that satisfies the equation $x^2 + 10x + 25 = 0$

Was Ben correct?
Show working to justify your answer

Solve Quadratics Using the Formula

$ax^2 + bx + c = 0$	$x_{1,2} = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

1) Solve the equation $x^2 + 4x + 1 = 0$
Give your answers correct to 3 decimal places.

2) Solve the equation $x^2 + 8x + 6 = 0$
Give your answers correct to 3 significant figures.

3) Solve the equation $x^2 - 3x - 2 = 0$
Give your answers correct to 3 significant figures.

4) Solve the equation $x^2 - 7x + 2 = 0$
Give your answers correct to 3 significant figures.

5) Solve the equation $2x^2 + 6x - 1 = 0$
Give your answers correct to 3 significant figures.

6) Solve the equation $3x^2 - 2x - 20 = 0$
Give your answers correct to 3 significant figures.

7) Solve the equation $x^2 - 14x - 161.25 = 0$

8) Solve the equation $17x^2 - 92x - 206 = 0$
Give your answers correct to 3 significant figures.

9) $x^2 + 10x = 300$
Find the positive value of x.
Give your answer correct to 3 significant figures.

10) $(x + 2)(x - 3) = 1$

a) Show that $x^2 - x - 7 = 0$

b) Solve the equation $x^2 - x - 7 = 0$
Give your answers correct to 3 significant figures.

Completing the Square

1) Show that if $y = x^2 + 8x - 3$
then $y \geq -19$ for all values of x.

2) Show that if $y = x^2 - 10x + 30$
then $y \geq 5$ for all values of x.

3) The expression $x^2 + 4x + 10$ can be written in the form
$(x + p)^2 + q$ for all values of x.
Find the values of p and q.

4) Given that $x^2 - 6x + 17 = (x - p)^2 + q$ for all values of x,
find the value of p and the value of q.

5) For all values of x,

$$x^2 + 6x = (x + p)^2 + q$$

a) Find the values of p and q.

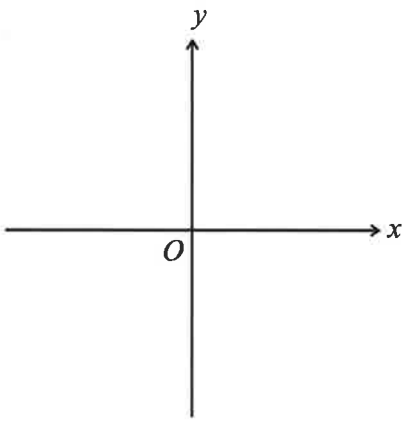
b) Find the minimum value of $x^2 + 6x$.

6) For all values of x,

$$x^2 - 8x - 5 = (x - p)^2 + q$$

a) Find the value of p and the value of q.

b) On the axes, sketch the graph of $y = x^2 - 8x - 5$.



c) Find the coordinate of the minimum point on the graph of $y = x^2 - 8x - 5$.

7) The expression $10x - x^2$ can be written in the form $p - (x - q)^2$ for all values of x.

a) Find the values of p and q.

b) The expression $10x - x^2$ has a maximum value.

(i) Find the maximum value of $10x - x^2$.

(ii) State the value of x for which this maximum value occurs.

Simultaneous Equations With a Quadratic

1) Solve these simultaneous equations.

$y = x$

$y = x^2 - 6$

2) Solve these simultaneous equations.

$y = x^2 - 4$

$y = 3x$

3) Solve these simultaneous equations.

$y = x^2 - x - 13$

$y = x + 2$

4) Solve these simultaneous equations.

$y = x^2 - 35$

$x - y = 5$

5) Solve these simultaneous equations.

$x^2 + y^2 = 26$

$y + 6 = x$

6) Sarah said that the line $y = 7$ cuts the curve $x^2 + y^2 = 25$ at two points.

a) By eliminating y show that Sarah is **not** correct.

b) By eliminating y, find the solutions to the simultaneous equations

$x^2 + y^2 = 25$

$y = 3x - 9$

Quadratic Inequalities

1. Solve the inequality $x^2 + 6x + 8 < 0$

2. Solve the inequality $x^2 + 2x - 35 > 0$

3. Solve the inequality $x^2 - 9x + 14 \leq 0$

4. Solve the inequality $x^2 - x - 30 \geq 0$

5. Solve the inequality $x^2 > 4(8 - x)$

6. Solve the inequality $3x^2 - 5x - 1 < 4x^2 + 7x + 19$

7. Solve the inequality $2x^2 + 9x + 10 > 0$

8. Solve the inequality $7x^2 - 22x + 16 \leq 0$

9. Find the set of values of x for which $x^2 - 2x - 24 < 0$ **and** $12 - 5x \geq x + 9$

10. Find the set of values of x for which $x^2 - 100 > 0$ **and** $x^2 + 8x - 105 > 0$

Proof

Question 1

w, x and y are three integers.

w is 2 less than x

y is 2 more than x

Prove that $wy + 4 = x^2$

[3 marks]

Question 2

c is a positive integer.

Prove that $\frac{6c^3 + 30c}{3c^2 + 15}$ is an even number.

[3 marks]

Question 3

Prove that $\frac{3n + 5}{3n} - \frac{n}{n - 1} \equiv \frac{2n - 5}{3n(n - 1)}$

[3 marks]

Question 4

Prove that $x^2 + x + 1$ is always positive

Question 5

Prove that $5n - (2n + 3)(n + 1)$ is always negative.

[3 marks]

Question 6

n is a positive integer.

Prove that $n^2 + 3n + 2$ must be a multiple of 2.

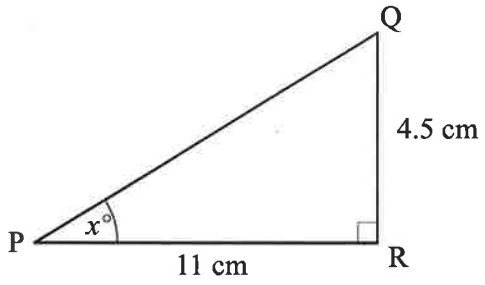
Question 9

Prove that the square of any odd number is always 1 more than a multiple of 8.

Trigonometry

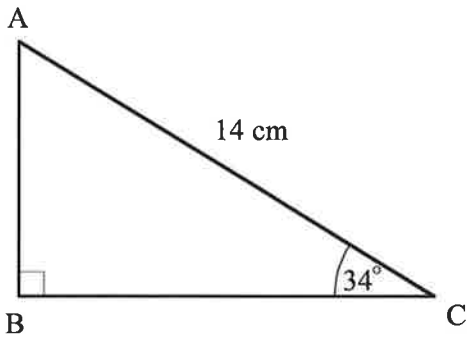
1) PQR is a right-angled triangle.
PR = 11 cm.
QR = 4.5 cm
Angle PRQ = 90°

Work out the value of x.
Give your answer correct to 1 decimal place.



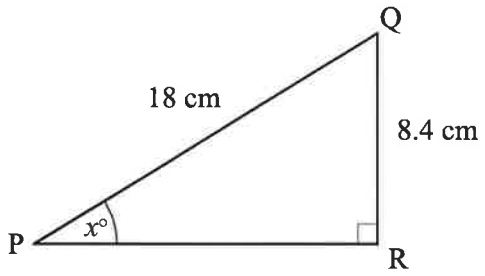
2) AC = 14 cm.
Angle ABC = 90°
Angle ACB = 34°

Calculate the length of BC.
Give your answer correct to
3 significant figures.



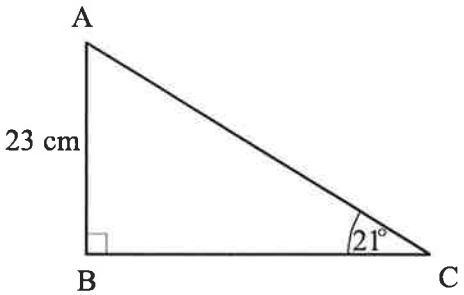
3) PQR is a right-angled triangle.
PQ = 18 cm.
QR = 8.4 cm
Angle PRQ = 90°

Work out the value of x.
Give your answer correct to 1 decimal place.



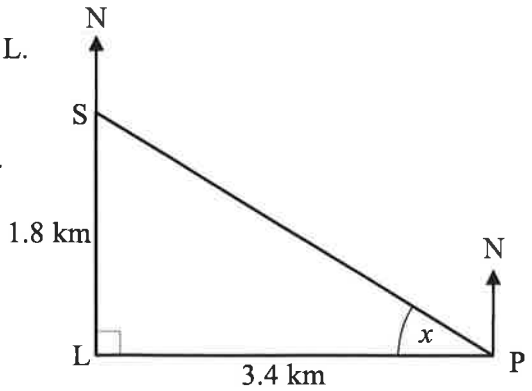
4) AB = 23 cm.
Angle ABC = 90°
Angle ACB = 21°

Calculate the length of AC.
Give your answer correct to
3 significant figures.



5) A lighthouse, L, is 3.4 km due West of a port, P.
A ship, S, is 1.8 km due North of the lighthouse, L.

Calculate the size of the angle marked x.
Give your answer correct to 3 significant figures.



Bearings by Trigonometry

1)

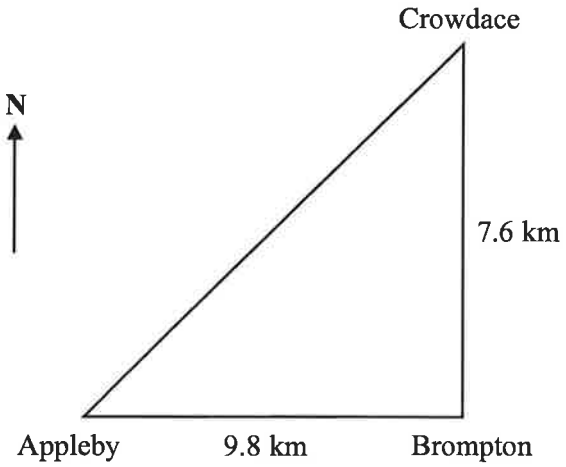


Diagram **NOT** accurately drawn.

Appleby, Brompton and Crowdace are three towns.

Appleby is 9.8 km due west of Brompton.
Brompton is 7.6 km due south of Crowdace.

a) Calculate the bearing of Crowdace from Appleby.
Give your answer correct to 1 decimal place.

b) Calculate the bearing of Appleby from Crowdace.
Give your answer correct to 1 decimal place.

2)

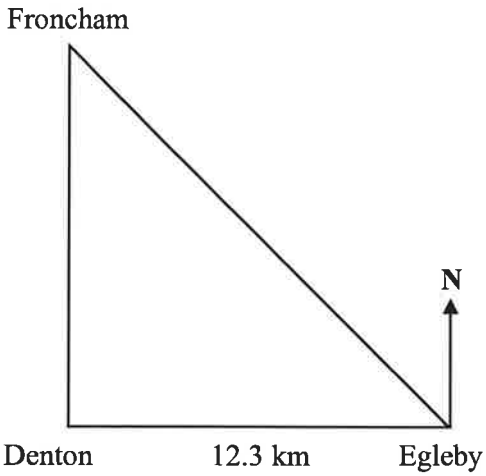


Diagram **NOT** accurately drawn.

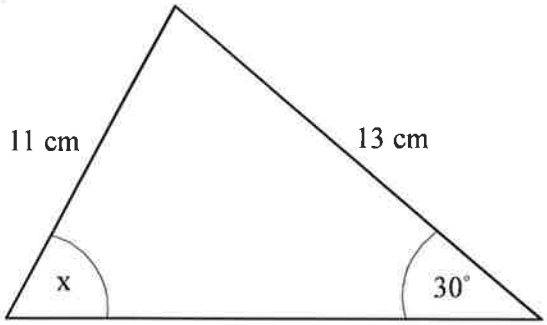
Denton, Egleby and Froncham are three towns.

Egleby is 12.3 km due East of Denton.
Froncham is due north of Denton and on a bearing of 320° from Egleby.

Calculate the distance between Froncham and Egleby.
Give your answer correct to 1 decimal place.

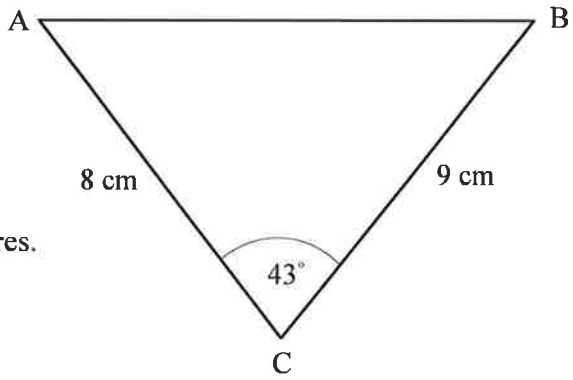
Sine and Cosine rules

1) Work out the size of the angle marked x.
Give your answer correct to one decimal place.



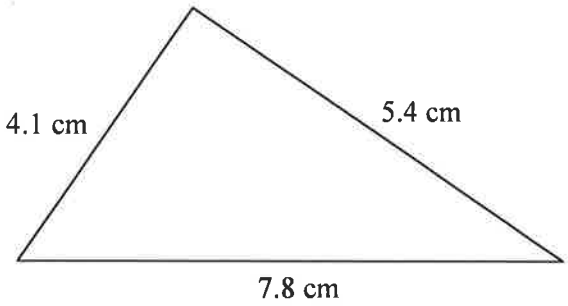
2) ABC is a triangle.
AC = 8 cm
BC = 9 cm
Angle ACB = 43°

Calculate the length of AB.
Give your answer correct to 3 significant figures.



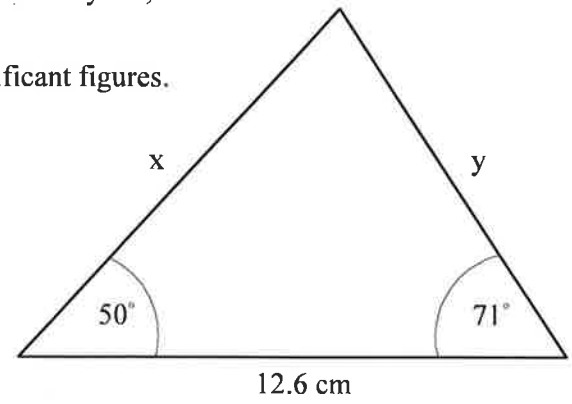
3) The lengths of the sides of a triangle are 4.1 cm, 5.4 cm and 7.8 cm.

Calculate the size of the largest angle of the triangle.
Give your answer correct to 1 decimal place.



4) Find the missing lengths, x cm and y cm,
in this triangle.

Give your answers to 3 significant figures.



Area of Triangles Using ½abSinC

1)

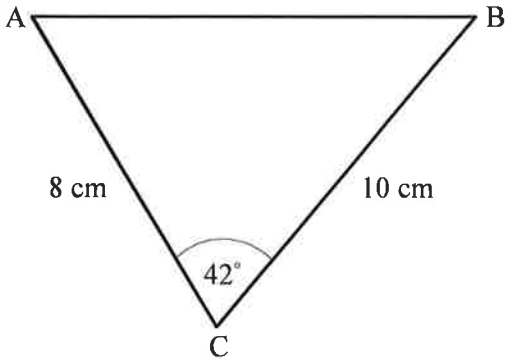


Diagram **NOT** accurately drawn.

ABC is a triangle.
AC = 8 cm.
BC = 10 cm
Angle ACB = 42°

Calculate the area of triangle ABC.
Give your answer correct to 3 significant figures.

2)

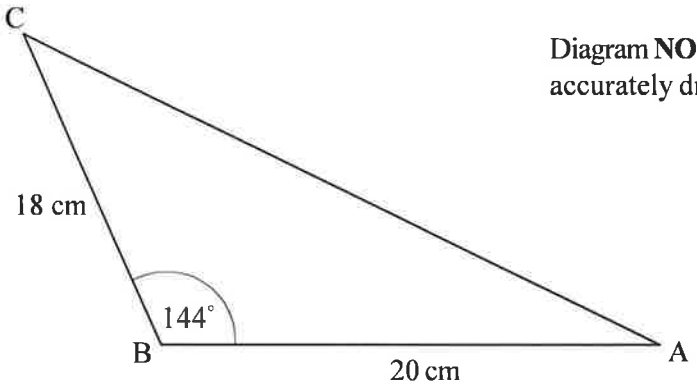


Diagram **NOT** accurately drawn.

ABC is a triangle.
AB = 20 cm.
BC = 18 cm
Angle ABC = 144°

Calculate the area of triangle ABC.
Give your answer correct to 3 significant figures.

3)

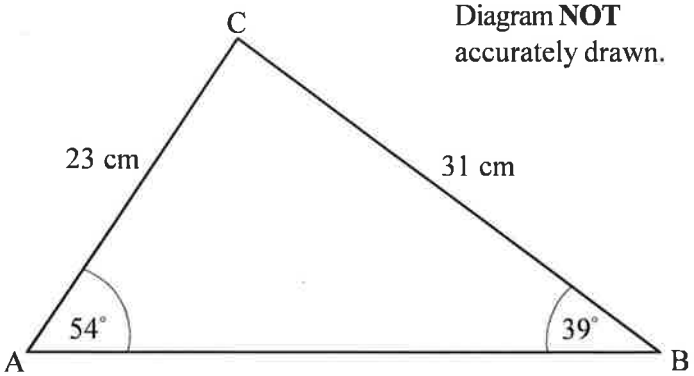


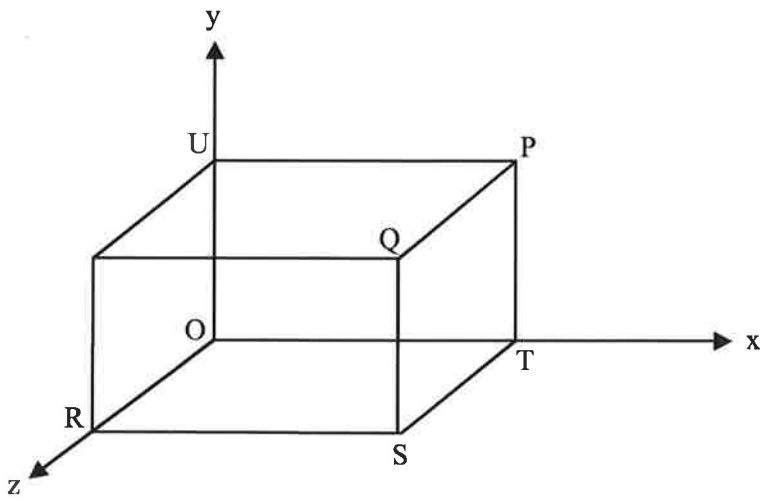
Diagram **NOT** accurately drawn.

ABC is a triangle.
AC = 23 cm.
BC = 31 cm
Angle BAC = 54°
Angle ABC = 39°

Calculate the area of triangle ABC.
Give your answer correct to 3 significant figures.

3-D Coordinates

1) A cuboid lies on the coordinate axes.



The point Q has coordinates (5, 3, 4)

a) Write down the coordinates of the point P

b) Write down the coordinates of the point T

c) Write down the coordinates of the point S

d) Write down the coordinates of the point R

e) Write down the coordinates of the point U

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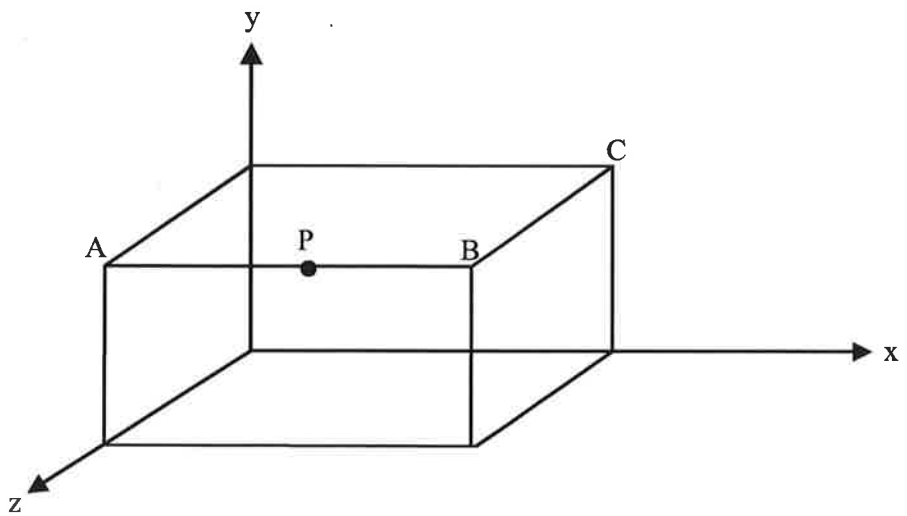
2) A cuboid lies on the coordinate axes.



Point P lies half way between A and B and has coordinates (3, 4, 5)

a) Write down the coordinates of B.

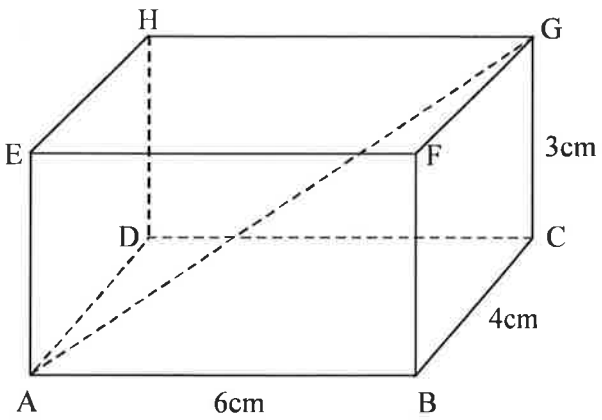
b) Write down the coordinates of C.

Pythagoras in 3 Dimensions

1) The diagram shows a box in the shape of a cuboid.
AB = 6cm, BC = 4cm, CG = 3cm

A string runs diagonally across the box from A to G.

Calculate the length of the string AG.
Give your answer correct to 3 significant figures.



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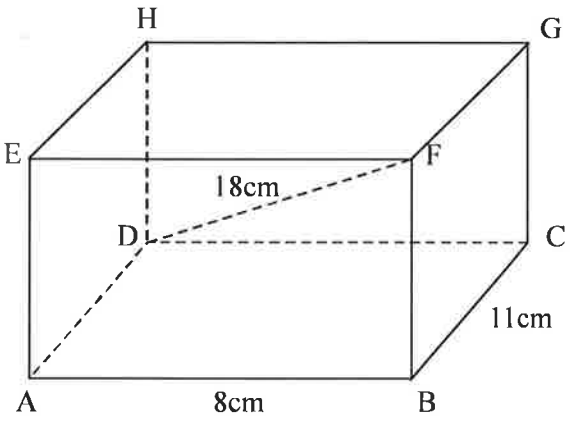
2) The diagram shows a box in the shape of a cuboid.
AB = 8cm, BC = 11cm

A string runs diagonally across the box from D to F and is 18cm long.

Calculate the length AE.
Give your answer correct to 3 significant figures.



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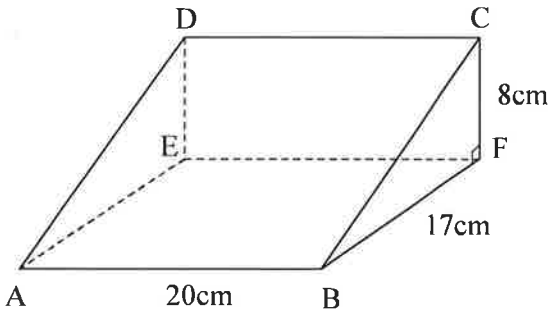
3) The diagram shows a wedge in the shape of a prism.
Angle BFC is a right angle.

String runs diagonally across the wedge from A to C.

Calculate the length AC
Give your answer correct to 3 significant figures.



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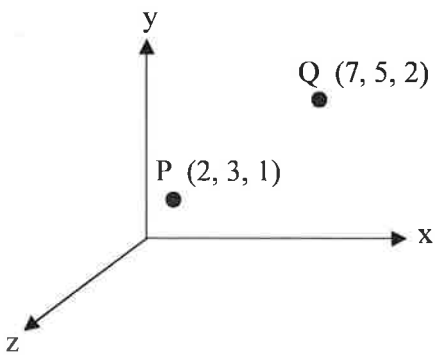
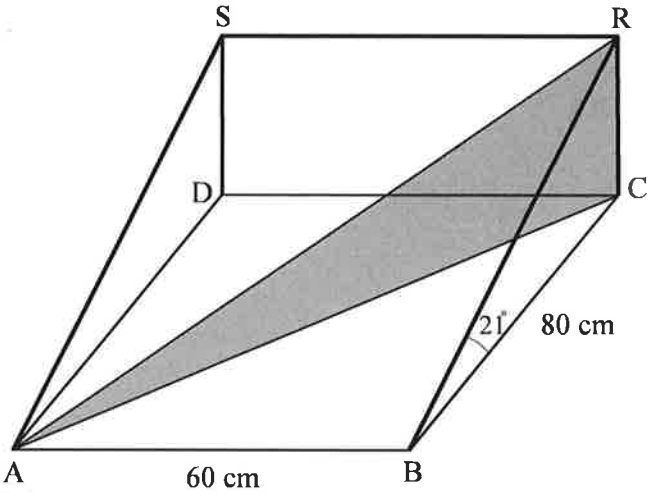
4) Two points, P and Q, lie on coordinate axes.

Find the distance PQ to 1 decimal place.



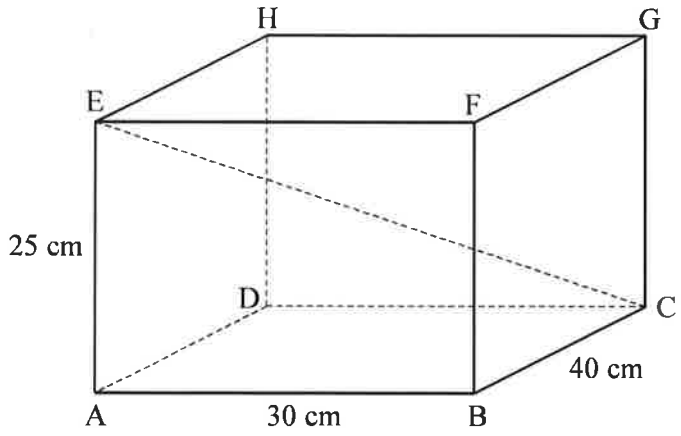
Trigonometry in 3 Dimensions

1) The diagram shows a wedge.
The base of the wedge is a horizontal rectangle measuring 80 cm by 60 cm.
The sloping face ABRS makes an angle of 21° to the horizontal.



Calculate the angle that AR makes with the horizontal plane ABCD.
Give your answer correct to 1 decimal place.

2) The diagram shows a box in the shape of a cuboid.
A string runs diagonally across the box from C to E.

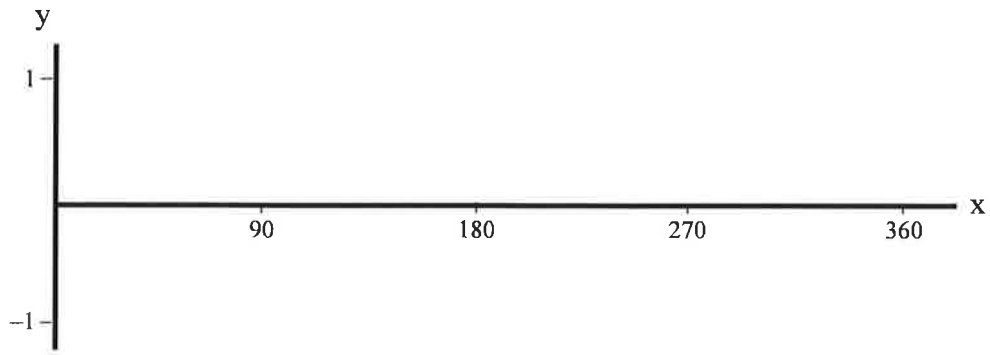


a) Work out the length of the string CE.
Give your answer correct to 1 decimal place.

b) Work out the angle between the string CE and the horizontal plane ABCD.
Give your answer correct to 1 decimal place.

Graphs of Trigonometric Functions - 1 of 2

1) On the axes below below, draw a sketch-graph to show $y = \sin x$

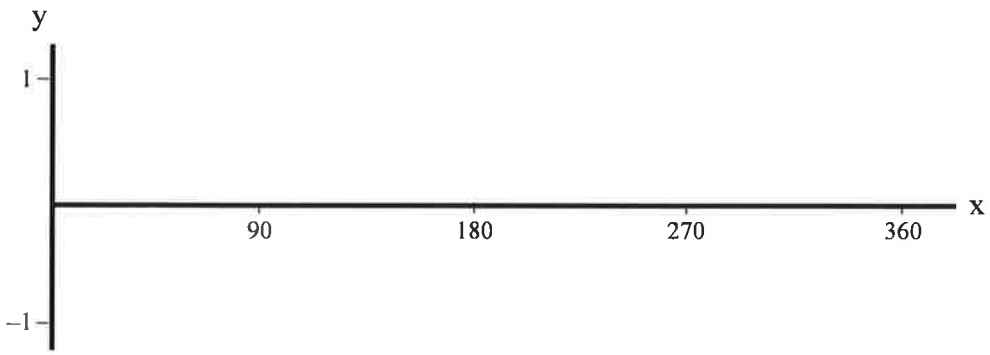


Given that $\sin 30° = 0.5$, write down the value of:

(i) $\sin 150°$

(ii) $\sin 330°$

2) On the axes below, draw a sketch-graph to show $y = \cos x$



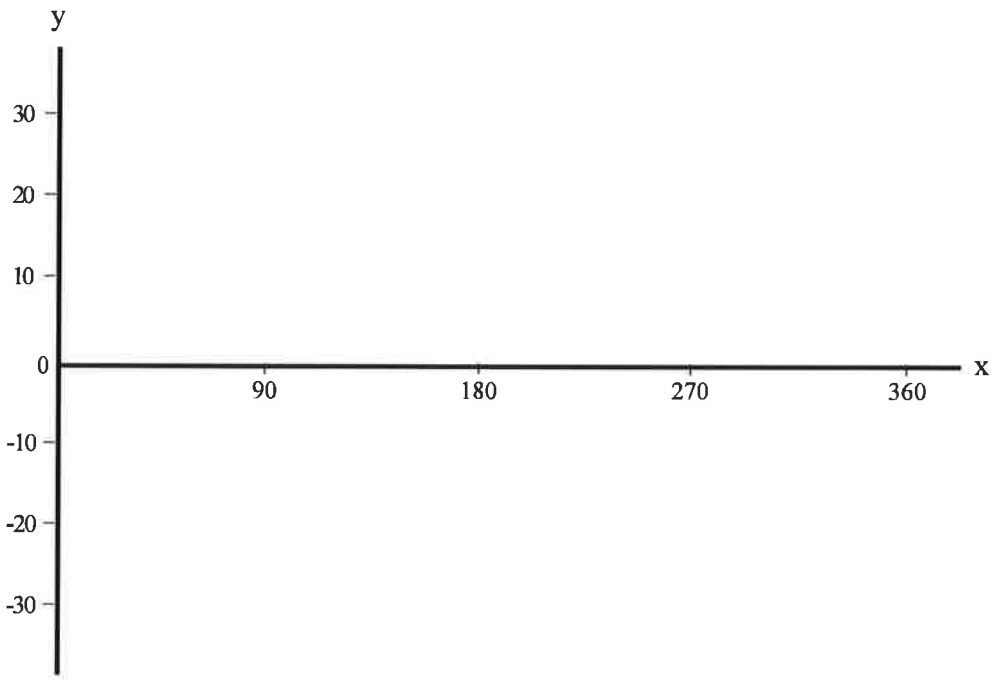
Given that $\cos 60° = 0.5$, write down the value of:

(i) $\cos 120°$

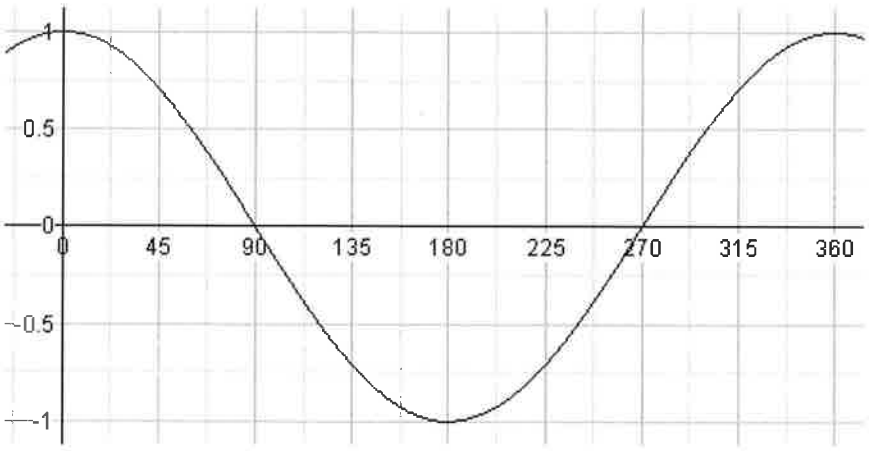
(ii) $\cos 240°$

Graphs of Trigonometric Functions - 2 of 2

1) On the axes below, draw a sketch-graph to show $y = \tan x$



2) Here is the graph of the curve $y = \cos x$ for $0 \leqslant x \leqslant 360^\circ$.

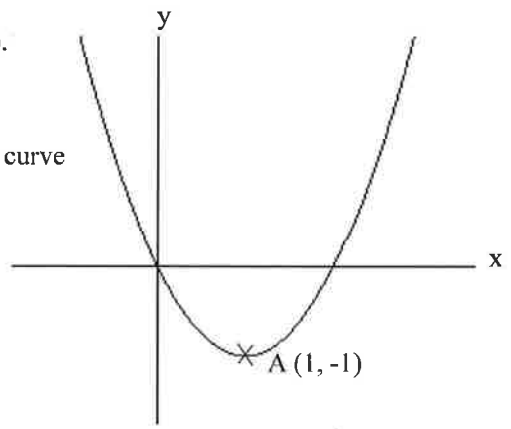


a) Use the graph to solve $\cos x = 0.75$ for $0 \leqslant x \leqslant 360^\circ$

b) Use the graph to solve $\cos x = -0.75$ for $0 \leqslant x \leqslant 360^\circ$

Transformations of Functions

1) This is a sketch of the curve with equation $y = f(x)$.
It passes through the origin O.
The only vertex of the curve is at A (1, -1)



a) Write down the coordinates of the vertex of the curve with equation

(i) $y = f(x - 3)$

(ii) $y = f(x) - 5$

(iii) $y = -f(x)$

(iv) $y = f(2x)$

b) The curve $y = x^2$ has been translated to give the curve $y = f(x)$.
Find $f(x)$ in terms of x.

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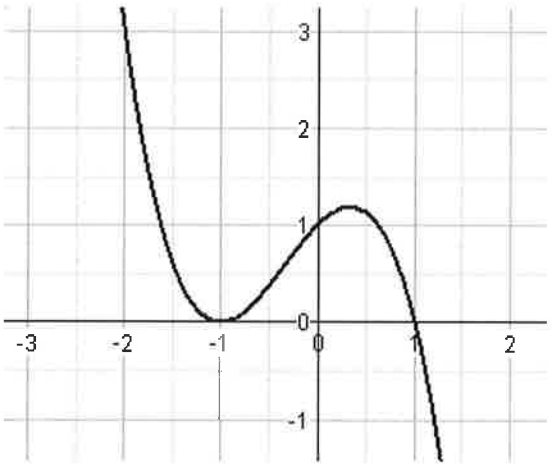
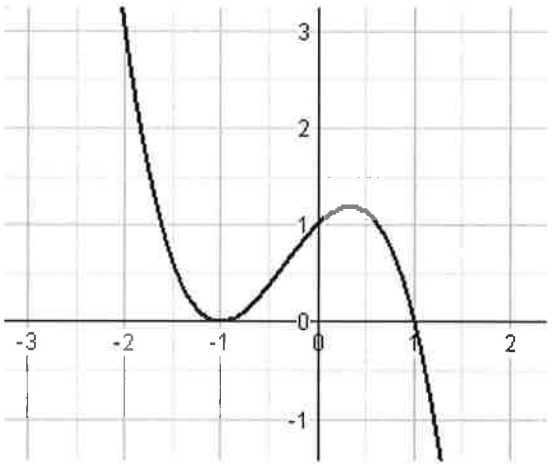
2) The graph of $y = f(x)$ is shown on the grids.

a) On this grid, sketch the graph of $y = f(x - 1)$



b) On this grid, sketch the graph of $y = 2f(x)$



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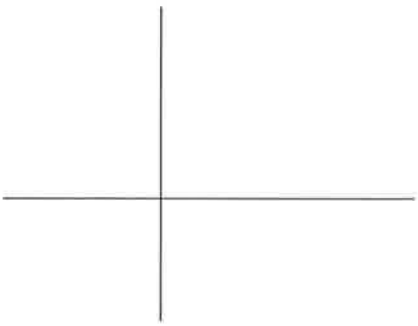
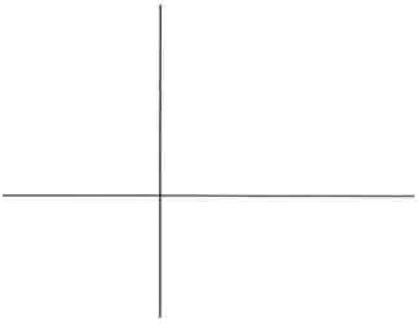
3) Sketch the graph of $y = (x - 2)^2 + 3$
State the coordinates of the vertex.

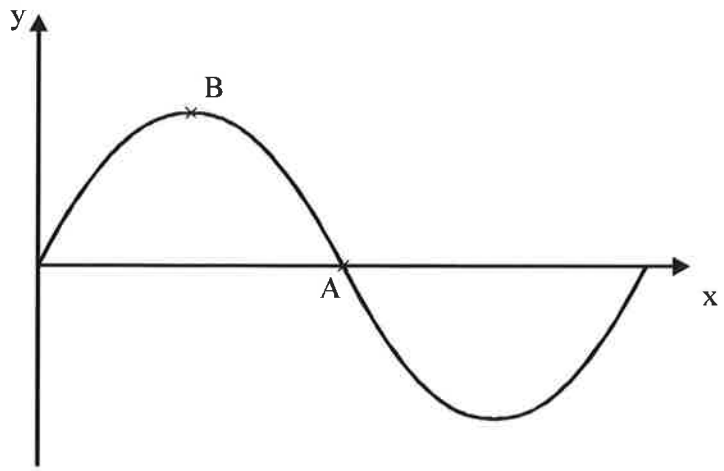


4) Sketch the graph of $y = x^2 + 4x - 1$
State the coordinates of the vertex and the points at which the curve crosses the x - axis.



Transformation of Trig. Functions

1) The diagram below shows the graph of $y = 2\sin x$, for values of x between 0 and 360°.



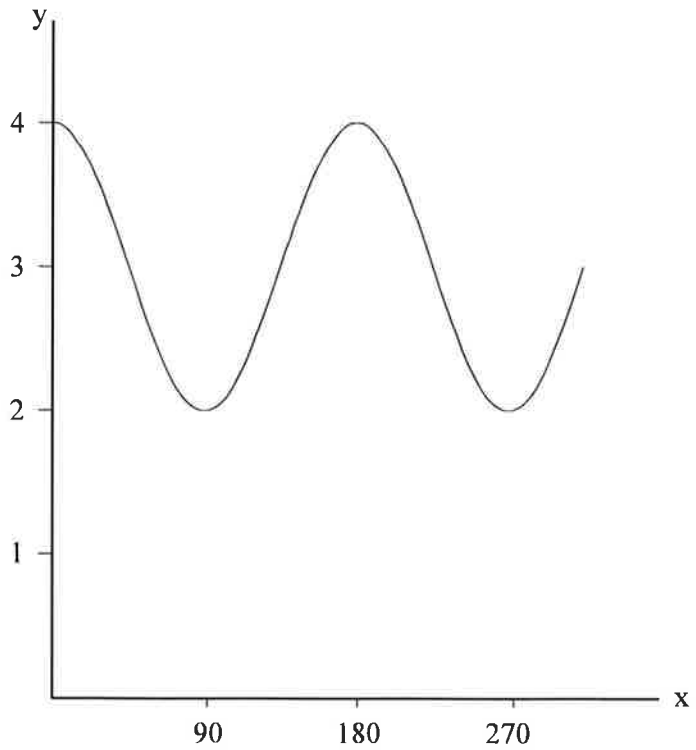
The curve cuts the x axis at the point A.
The graph has a maximum at the point B.

a) (i) Write down the coordinates of A.
(ii) Write down the coordinates of B.

b) On the same diagram, sketch the graph of $y = 2\sin x + 1$ for values of x between 0 and 360°.

2) The diagram below shows the graph of $y = \cos \mathbf{a}x + \mathbf{b}$, for values of x between 0 and 300°.

Work out the values of **a** and **b**.



Iterations

1 (a) Show that the equation $x^3 + 2x = 1$ has a solution between $x = 0$ and $x = 1$

(b) Show that the equation $x^3 + 2x = 1$ can be rearranged to give $x = \frac{1}{2} - \frac{x^3}{2}$

(c) Starting with $x_0 = 0$, use the iteration formula $x_{n+1} = \frac{1}{2} - \frac{x_n^3}{2}$ twice to find an estimate for the solution of $x^3 + 2x = 1$

2 (a) Show that the equation $3x - x^3 = -11$ has a solution between $x = 2$ and $x = 3$

(b) Show that the equation $3x - x^3 = -11$ can be rearranged to give

$$x = \sqrt[3]{3x + 11}$$

(c) Starting with $x_0 = 3$, use the iteration formula $x_{n+1} = \sqrt[3]{3x_n + 11}$ three times to find an estimate for the solution of $3x - x^3 = -11$

3 (a) Show that the equation $20 - x^3 - 7x^2 = 0$ can be rearranged to give

(b) Using $x_{n+1} = \frac{20}{x_n^2} - 7$ with $x_0 = -9$

find the values of x_1, x_2 and x_3

(c) Explain what the values of x_1, x_2 and x_3 represent

Direct and Inverse Proportion

1) x is directly proportional to y.
When $x = 21$, then $y = 3$.

a) Express x in terms of y.

b) Find the value of x when y is equal to:
(i) 1 (ii) 2 (iii) 10

2) a is inversely proportional to b.
When $a = 12$, then $b = 4$.

a) Find a formula for a in terms of b.

b) Find the value of a when b is equal to:
(i) 1 (ii) 8 (iii) 10

c) Find the value of b when a is equal to:
(i) 4 (ii) 24 (iii) 3.2

3) The variables u and v are in inverse proportion to one another.
When $u = 3$, then $v = 8$.

Find the value of u when $v = 12$.

4) p is directly proportional to the square of q.
$p = 75$ when $q = 5$

a) Express p in terms q.

b) Work out the value of p when $q = 7$.

c) Work out the positive value of q when $p = 27$.

5) y is directly proportional to x^2.
When $x = 3$, then $y = 36$.

a) Express y in terms of x.

z is inversely proportional to x.
When $x = 4$, $z = 2$.

b) Show that $z = c\,y^n$, where c and n are numbers and $c > 0$.
You must find the values of c and n.

Surds

$\sqrt{25}$ is not a surd because it is equal to exactly 5.

$\sqrt{3}$ is a surd because you can only ever approximate the answer.

We don't like surds as denominators.
When we **rationalise** the denominator it means that we transfer the surd expression to the numerator.

---

1) Simplify the following:

a) $\sqrt{7} \times \sqrt{7}$

b) $\sqrt{3} \times \sqrt{3}$

c) $\sqrt{20}$

d) $\sqrt{24}$

e) $\sqrt{72}$

f) $\sqrt{200}$

g) $\sqrt{\frac{2}{25}}$

2) Simplify the following:

a) $\sqrt{2} \times \sqrt{18}$

b) $\sqrt{8} \times \sqrt{32}$

c) $\sqrt{99} \times \sqrt{22}$

d) $\sqrt{45} \times \sqrt{20}$

e) $\sqrt{18} \times \sqrt{128}$

f) $\sqrt{28} \times \sqrt{175}$

3) Expand and simplify where possible:

a) $\sqrt{3}(3-\sqrt{3})$

b) $\sqrt{2}(6+2\sqrt{2})$

c) $\sqrt{7}(2+3\sqrt{7})$

d) $\sqrt{2}(\sqrt{32}-\sqrt{8})$

4) Expand and simplify where possble:

a) $(1+\sqrt{2})(1-\sqrt{2})$

b) $(3+\sqrt{5})(2-\sqrt{5})$

c) $(\sqrt{3}+2)(\sqrt{3}+4)$

d) $(\sqrt{5}-3)(\sqrt{5}+1)$

e) $(2+\sqrt{7})(2-\sqrt{7})$

f) $(\sqrt{6}-3)^2$

5) Rationalise the denominator, simplifying where possible:

a) $\frac{3}{\sqrt{2}}$

b) $\frac{2}{\sqrt{2}}$

c) $\frac{3\sqrt{2}}{\sqrt{7}}$

d) $\frac{\sqrt{5}}{\sqrt{10}}$

e) $\frac{1}{4\sqrt{8}}$

f) $\frac{\sqrt{15}}{\sqrt{3}}$

g) $\frac{1}{\sqrt{27}}$

6) $3 \times \sqrt{27} = 3^n$

Find the value of n

7) Express $8\sqrt{8}$ in the form $m\sqrt{2}$ where m is an integer.

8) Rationalise the denominator of $\frac{1}{8\sqrt{8}}$ giving the answer in the form $\frac{\sqrt{2}}{p}$

9) Work out the following, giving your answer in its simplest form:

a) $\frac{(5+\sqrt{3})(5-\sqrt{3})}{\sqrt{22}}$

b) $\frac{(4-\sqrt{5})(4+\sqrt{5})}{\sqrt{11}}$

c) $\frac{(3-\sqrt{2})(3+\sqrt{2})}{\sqrt{14}}$

d) $\frac{(\sqrt{3}+1)^2}{\sqrt{3}}$

e) $\frac{(\sqrt{5}+3)^2}{\sqrt{20}}$

f) $\frac{(5-\sqrt{5})(2+2\sqrt{5})}{\sqrt{20}}$

Fractional and Negative Indices

$a^x \times a^y = a^{x+y}$	$\frac{a^x}{a^y} = a^{x-y}$	$(a^x)^y = a^{xy}$	
$a^0 = 1$	$a^{-x} = \frac{1}{a^x}$	$a^{\frac{x}{y}} = (\sqrt[y]{a})^x$	$a^{-\frac{x}{y}} = \frac{1}{(\sqrt[y]{a})^x}$

1) Simplify

a) $(p^5)^5$ c) $x^5 \div x^2$ e) $(m^{-5})^{-2}$

b) $k^3 \times k^2$ d) $(p^2)^{-3}$ f) $(3xy^2)^3$

2) Without using a calculator, find the exact value of the following.

a) $4^0 \times 4^2$ c) $7^5 \div 7^3$ e) $(8^5)^0$

b) $5^4 \times 5^{-2}$ d) $\frac{6^7}{6^6}$ f) $(2^3)^2$

3) Work out each of these, leaving your answers as exact fractions when needed.

a) 4^0 e) 4^{-2} i) $49^{\frac{1}{2}}$ m) $49^{-\frac{1}{2}}$

b) 7^0 f) 8^{-1} j) $32^{\frac{2}{5}}$ n) $32^{-\frac{2}{5}}$

c) 25^0 g) 5^{-3} k) $27^{\frac{1}{3}}$ o) $27^{-\frac{1}{3}}$

d) 139^0 h) 10^{-5} l) $16^{\frac{3}{2}}$ p) $16^{-\frac{3}{2}}$

4) $5\sqrt{5}$ can be written in the form 5^n.
Find the value of n.

5) $2 \times \sqrt{8} = 2^m$
Find the value of m.

6) Find the value of x when
$\sqrt{125} = 5^x$

7) Find the value of y when
$\sqrt{128} = 2^y$

8) $a = 2^x$, $b = 2^y$

a) Express in terms of a and b

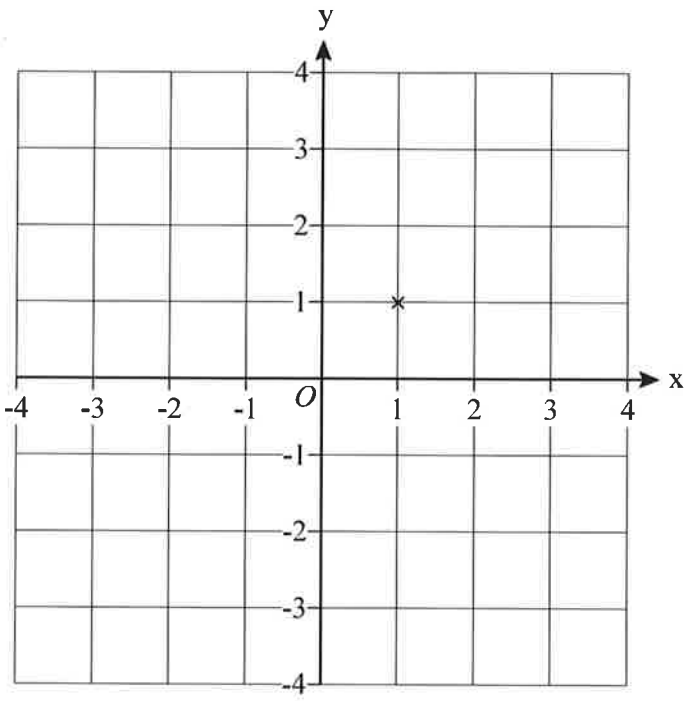
i) 2^{x+y} ii) 2^{2x} iii) 2^{x+2y}

$ab = 16$ and $2ab^2 = 16$

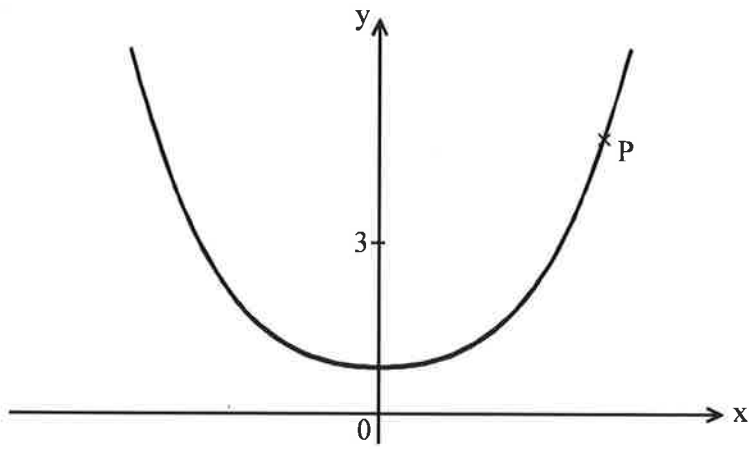
b) Find the value of x and the value of y.

Equations of Circles and Loci

1) Show that any straight line which passes through the point (1, 1) must intersect the curve with equation $x^2 + y^2 = 9$ at two points.



2)



The diagram shows a sketch of a curve.
The point P(x, y) lies on the curve.
The locus of P has the following property:

> The distance of the point P from the point (0, 3) is the same as the distance of the point P from the x-axis.

Show that $y = \frac{x^2 + 9}{6}$

Cubic and Reciprocal Functions

1) a) Complete this table of values for

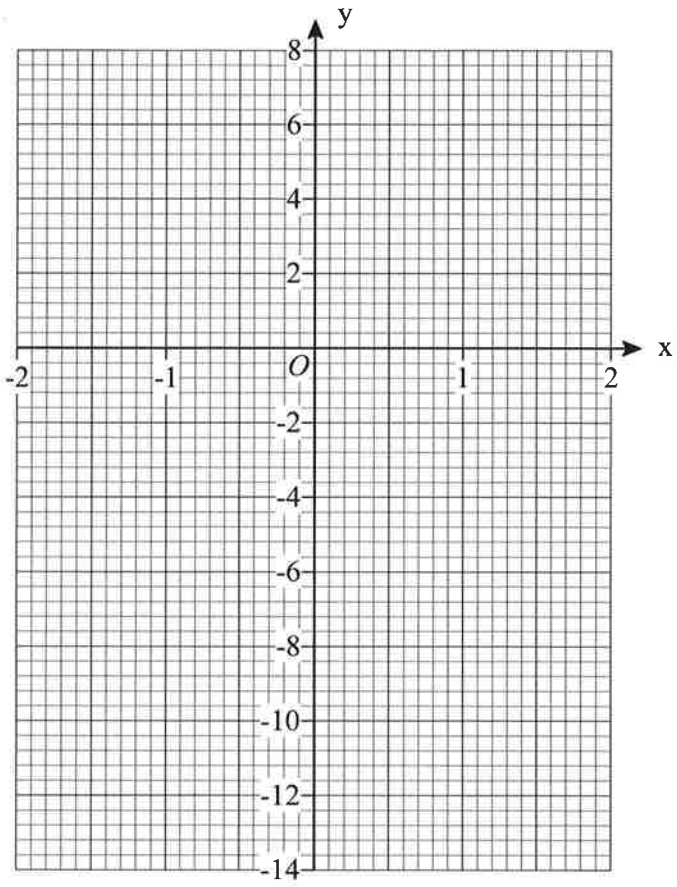
$y = x^3 + x - 4$

x	–2	–1	0	1	2
y	–14			–2	

b) On the grid, draw the graph of

$y = x^3 + x - 4$

c) Use the graph to find the value of x when y = 2



2) a) Complete this table of values for

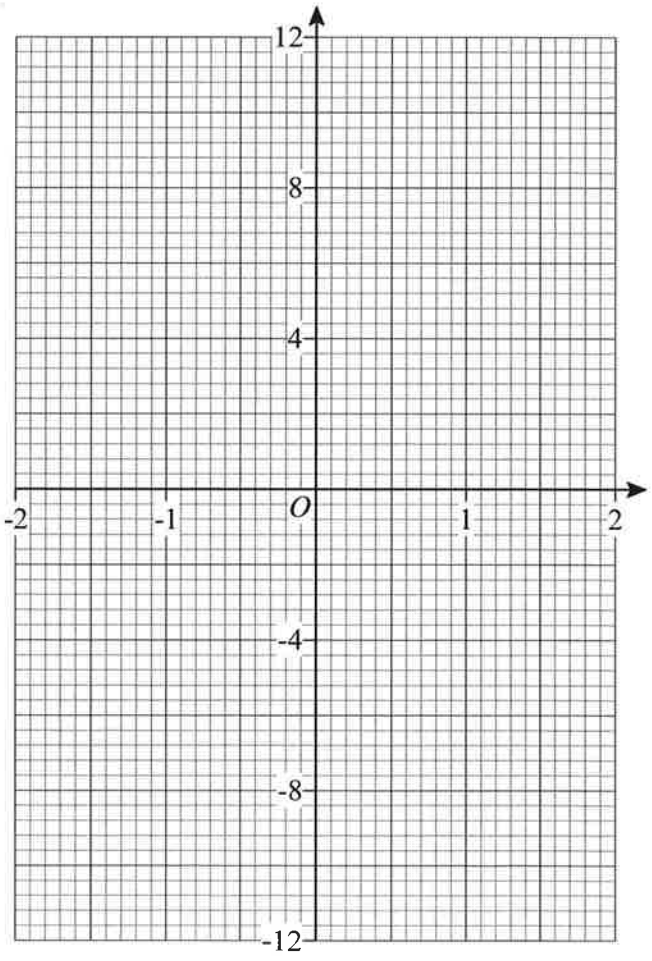
$y = x^3 + 2x$

x	–2	–1	0	1	2
y	–12		0		

b) On the grid, draw the graph of

$y = x^3 + 2x$

c) Use the graph to find the value of x when y = -6



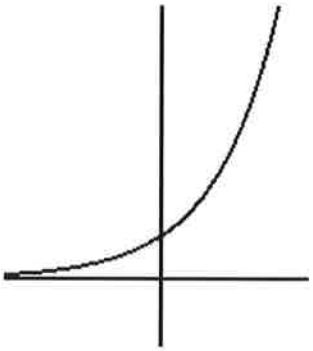
3) Sketch the graph of $y = 1 + \frac{1}{x}$ in your book.

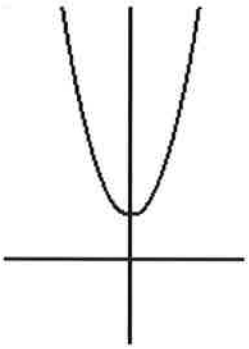
Recognise the Shapes of Functions

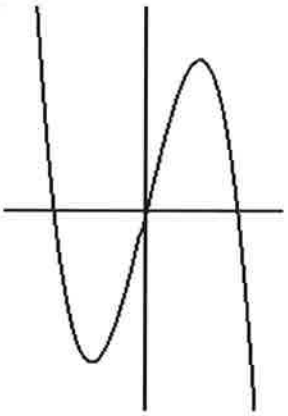
Match each of the functions below, with the correct sketch of its graph.

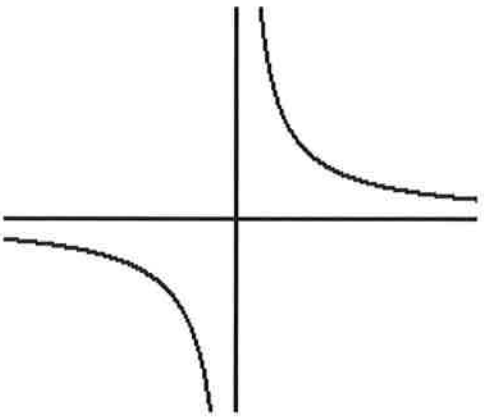
$y = 3x^3$ $\quad y = \frac{-2}{x}$ $\quad y = 3x - 1$ $\quad y = 2^x$

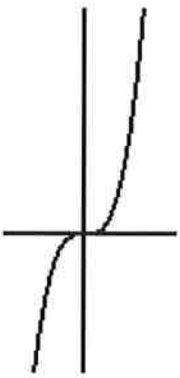
$y = 2x^2 + 1$ $\quad y = \frac{2}{x}$ $\quad y = 5x - x^3$ $\quad y = -2x^3$

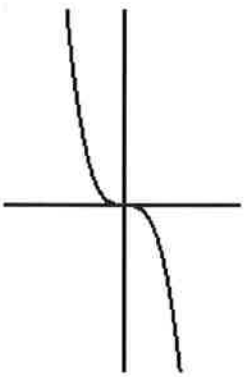


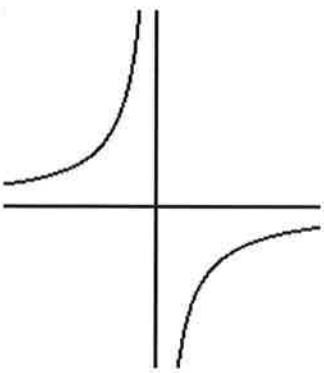


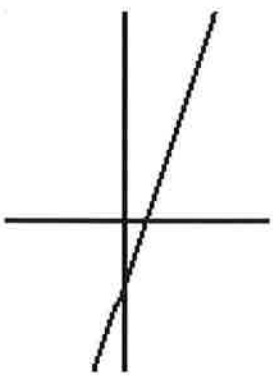






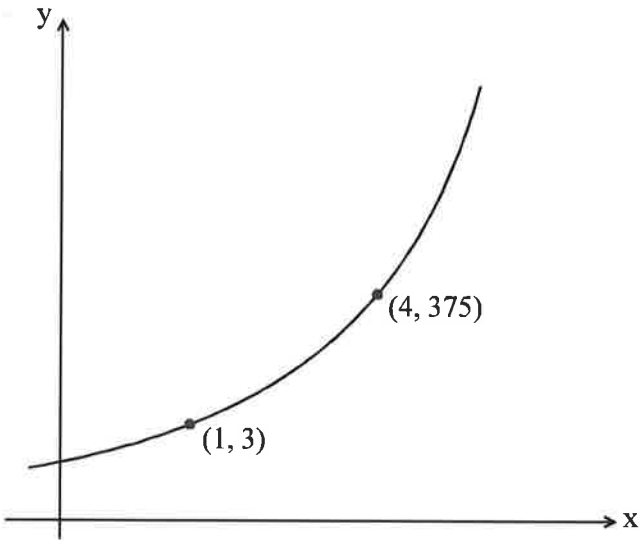






Graphs of Exponential Functions

1)



The sketch-graph shows a curve with equation $y = pq^x$.

The curve passes through the points (1, 3) and (4, 375).

Calculate the value of p and the value of q.

2) The graph shows the number of bacteria living in a petri dish.
The number N of bacteria at time t is given by the relation:

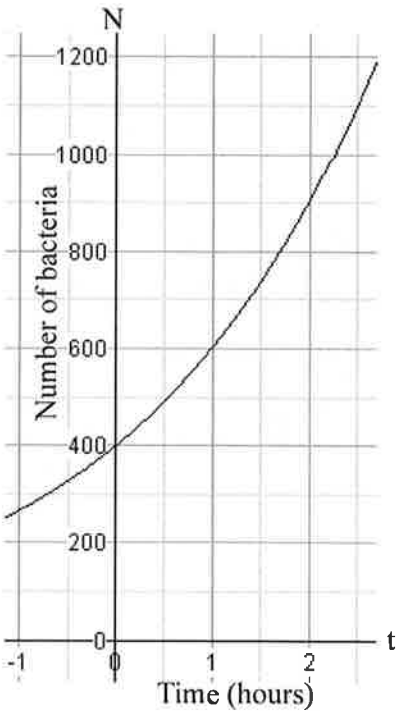
$N = a \times b^t$

The curve passes through the point (0, 400).

a) Use this information to show that a = 400.

The curve also passes through (2, 900).

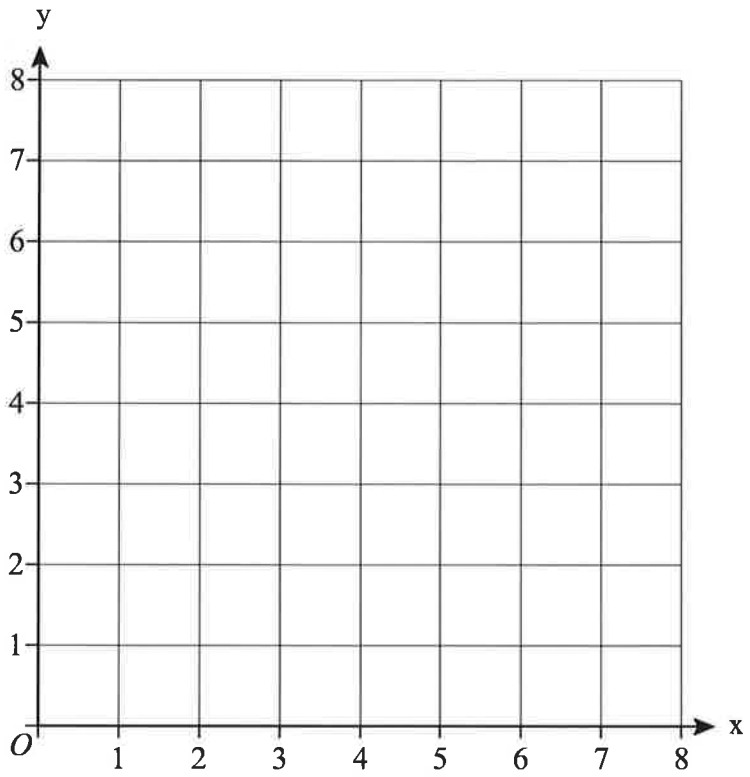
b) Use this information to find the value of b.



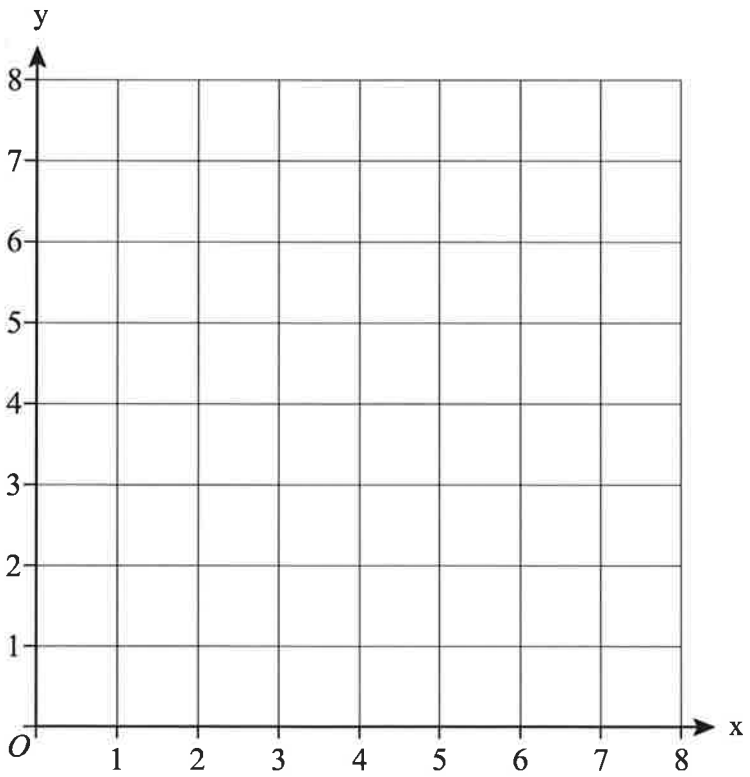
c) Work out the number of bacteria in the dish at time t = 3.

Regions

1) On the grid below, draw straight lines and use shading to show the region **R** that satisfies the inequalities $x \geqslant 1$ $y \geqslant x$ $x + y \leqslant 7$

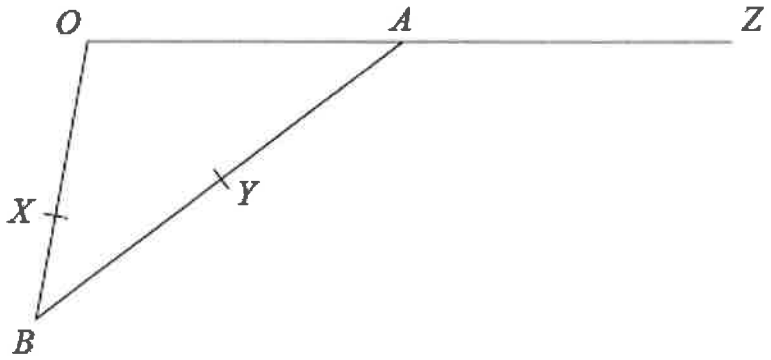


2) On the grid below, draw straight lines and use shading to show the region **R** that satisfies the inequalities $y \geqslant x + 1$ $y \leqslant 5$ $x \geqslant 1$



Vectors

1

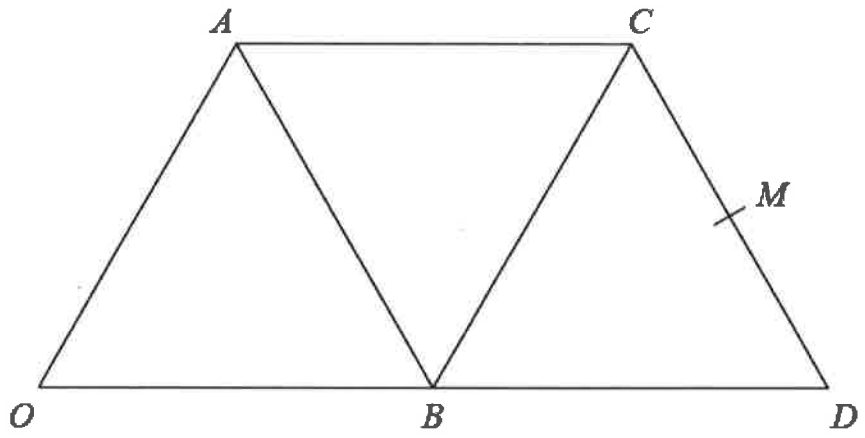


OAB is a triangle.
A is the midpoint of OZ
Y is the midpoint of AB
X is a point on OB

$\overrightarrow{OA} = \mathbf{a}$ $\overrightarrow{OX} = 2\mathbf{b}$ $\overrightarrow{XB} = \mathbf{b}$

Prove that XYZ is a straight line.

2



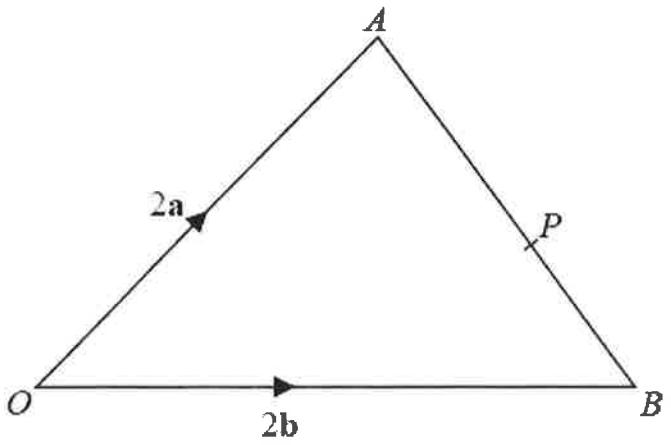
$OACD$ is a trapezium and $OACB$ is a parallelogram.
B is the midpoint of OD.
M is the midpoint of CD.

$\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$

Given that $\overrightarrow{BM} = k \times \overrightarrow{OC}$ where k is a scalar,

use a vector method to find the value of k.

3



OAB is a triangle.
P is the point on AB such that $AP : PB = 5 : 3$

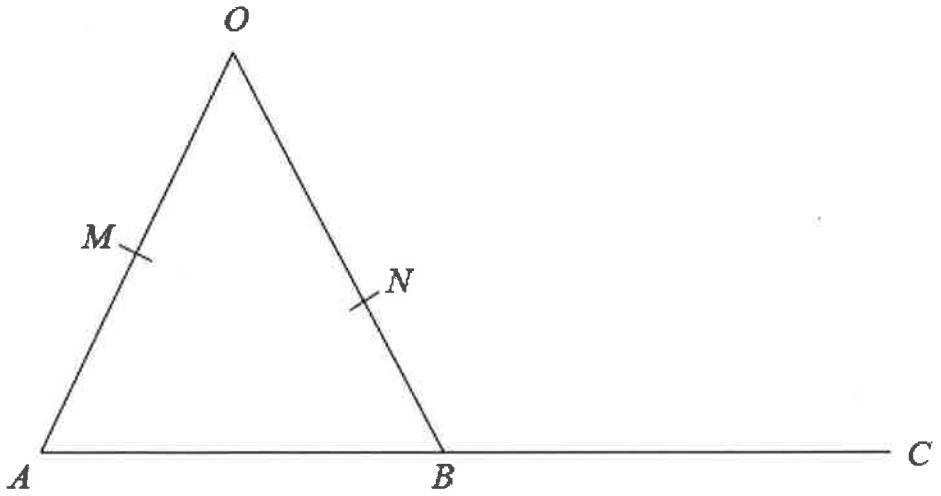
$\overrightarrow{OA} = 2\mathbf{a}$

$\overrightarrow{OB} = 2\mathbf{b}$

$\overrightarrow{OP} = k(3\mathbf{a} + 5\mathbf{b})$ where k is a scalar quantity.

Find the value of k.

4



OMA, ONB and ABC are straight lines.
M is the midpoint of OA.
B is the midpoint of AC.
$\overrightarrow{OA} = 6\mathbf{a}$ $\overrightarrow{OB} = 6\mathbf{b}$ $\overrightarrow{ON} = k\mathbf{b}$ where k is a scalar quantity.

Given that MNC is a straight line, find the value of k.

1)

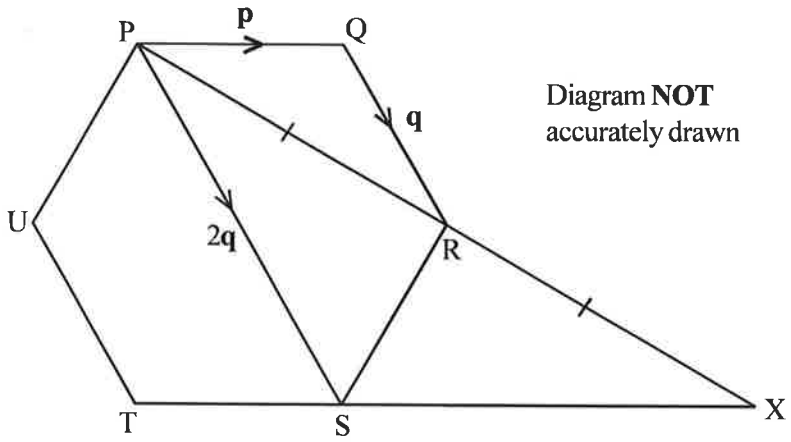


Diagram **NOT** accurately drawn

PQRSTU is a regular hexagon.

$\overrightarrow{PQ} = \mathbf{p}$ $\overrightarrow{QR} = \mathbf{q}$ $\overrightarrow{PS} = 2\mathbf{q}$

a) Find the vector PR in terms of **p** and **q**.

$\overrightarrow{PR} = \overrightarrow{RX}$

b) Prove that PQ is parallel to SX

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2)

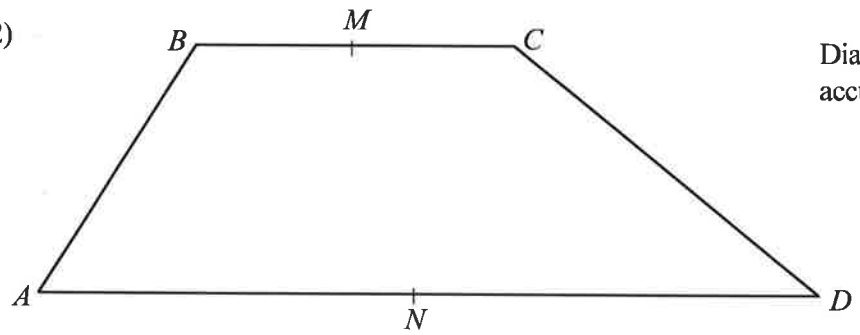


Diagram **NOT** accurately drawn

ABCD is a trapezium with *BC* parallel to *AD*.

$\overrightarrow{AB} = 3\mathbf{b}$ $\overrightarrow{BC} = 3\mathbf{a}$ $\overrightarrow{AD} = 9\mathbf{a}$

M is the midpoint of *BC* and N is the midpoint of *AD*.

a) Find the vector *MN* in terms of **a** and **b**.

X is the midpoint of *MN* and *Y* is the midpoint of *CD*.

b) Prove that *XY* is parallel to *AD*.

Averages From a Table

1) The number of pens in each pupil's pencil case in a classroom has been counted.
The results are displayed in a table.

Number of pens	Number of pupils
0	4
1	6
2	7
3	5
4	3
5	1

a) Work out the total number of pens in the classroom.

b) Write down the modal number of pens in a pencil case.

c) Work out the mean number of pens in a pencil case.

d) Work out the range of the number of pens in a pencil case.

2) Thomas is analysing the local football team.
He records the number of goals scored in each football match in the past twelve months.

Goals scored	Frequency
0	7
1	5
2	3
3	6
4	2
5	1
6	1

Thomas said that the mode is 7
Thomas is wrong.

a) Explain why.

b) Calculate the mean number of goals scored.

3) Tina recorded how long, in minutes, she watched TV for each day during a month.

Time (t in minutes)	Frequency
$10 < t \leqslant 20$	5
$20 < t \leqslant 30$	9
$30 < t \leqslant 45$	8
$45 < t \leqslant 60$	6
$60 < t \leqslant 90$	3

a) Find the class interval in which the median lies.

b) Work out an estimate for the mean amount of time Tina watched TV each day of this month.
Give your answer to the nearest minute.

Scatter Graphs

1) The scatter graph shows some information about the marks of six students.
It shows each student's marks in Maths and Science.

The table below shows the marks for four more students.

Maths	22	8	17	26
Science	30	12	24	24

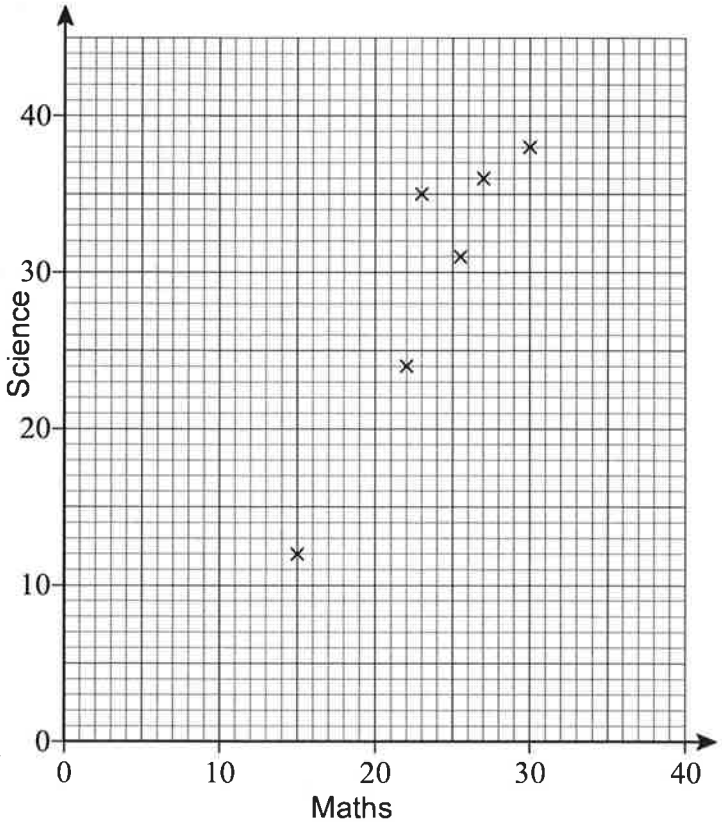
a) On the scatter graph, plot the information from the table.

b) Draw a line of best fit.

c) Describe the correlation between the marks in Maths and the marks in Science.

Another student has a mark of 18 in Science.

d) Use the line of best fit to estimate the mark in Maths of this student.



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2) The table below shows the average daily number of hours sleep of 10 children.

Age (years)	4	2	5	1	9	6	8	7	10	1.5
Number of hours sleep	14	13	12.5	15	10	12.5	10.8	12	11	14

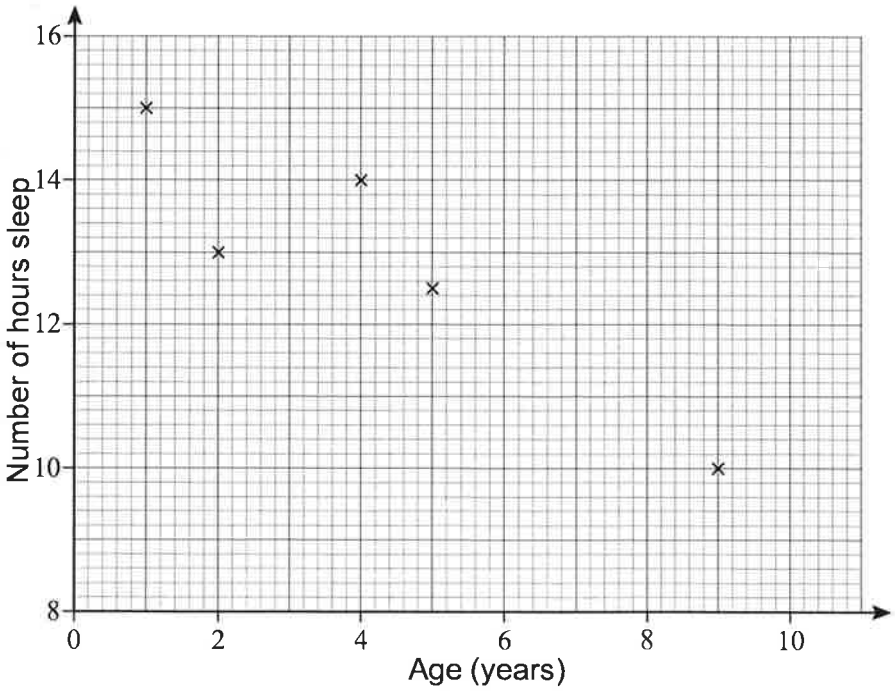
The first five results have been plotted on the scatter diagram.

a) Plot the next five points.

b) Draw a line of best fit.

c) Decribe the relationship between the age of the children and their number of hours sleep per day.

d) Use your scatter graph to estimate the number of hours sleep for a 3 year old child.



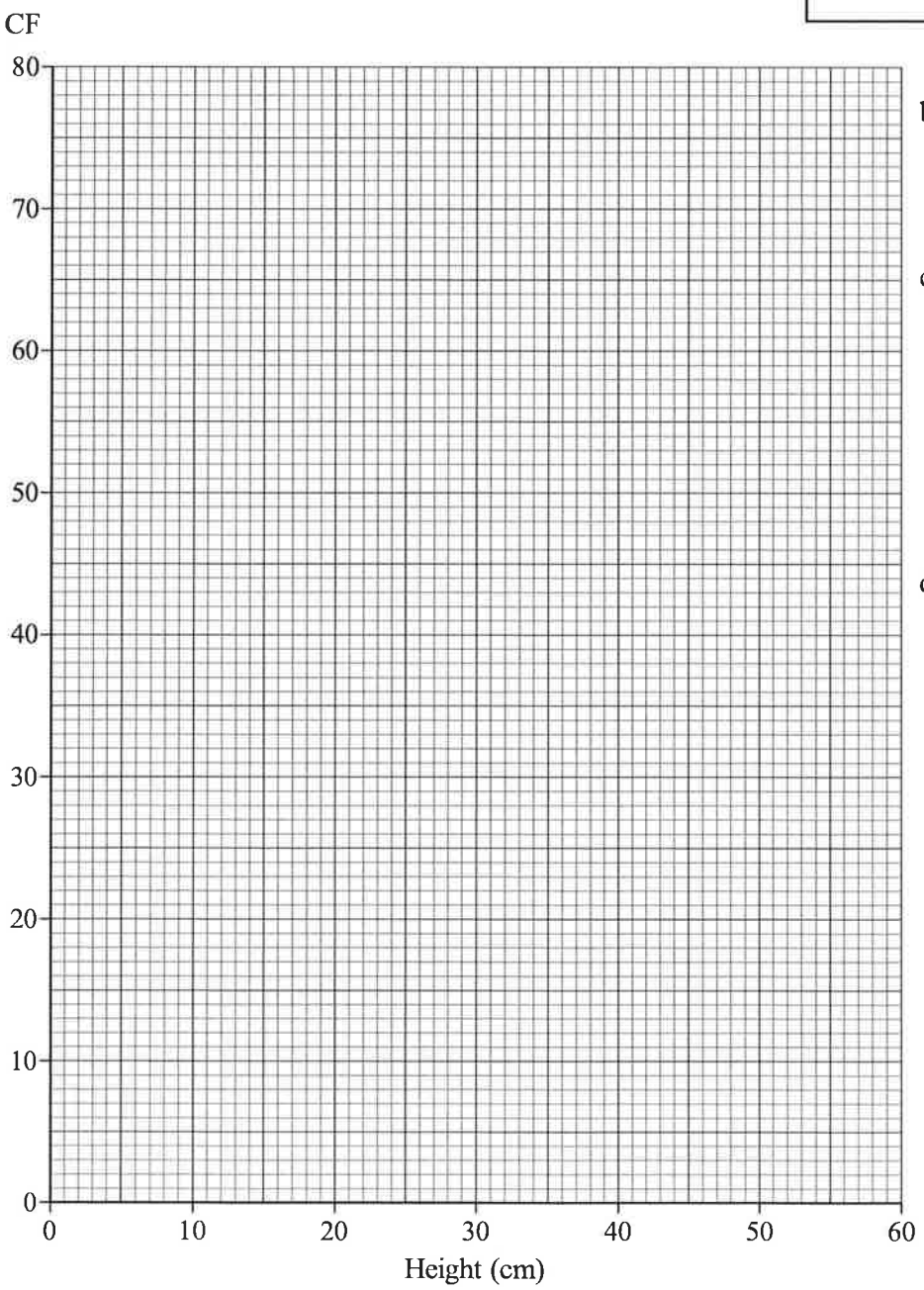
Cumulative Frequency

The heights of 80 plants were measured and can be seen in the table, below.

Height (cm)	Frequency
$0 < h \leqslant 10$	2
$10 < h \leqslant 20$	5
$20 < h \leqslant 30$	19
$30 < h \leqslant 40$	38
$40 < h \leqslant 50$	13
$50 < h \leqslant 60$	3

a) Complete the cumulative frequency table for the plants.

Height (cm)	Cumulative Frequency
$0 < h \leqslant 10$	2
$0 < h \leqslant 20$	
$0 < h \leqslant 30$	
$0 < h \leqslant 40$	
$0 < h \leqslant 50$	
$0 < h \leqslant 60$	



b) Draw a cumulative frequency graph for your table.

c) Use your graph to find an estimate for

(i) the median height of a plant.

(ii) the interquartile range of the heights of the plants.

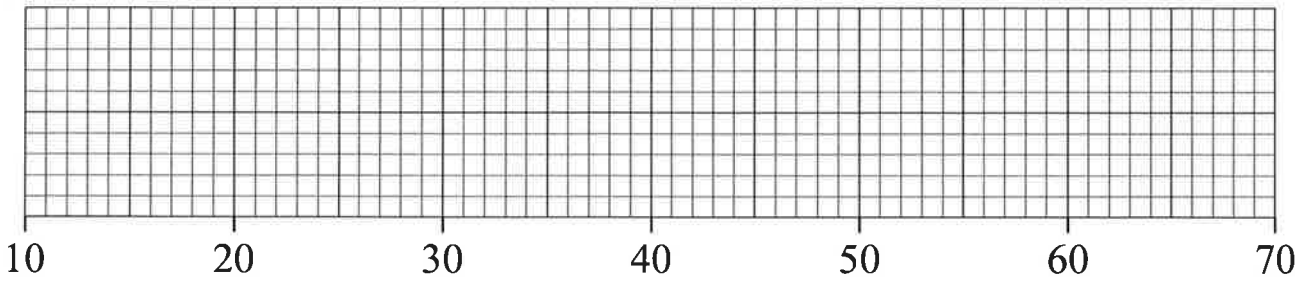
d) Use your graph to estimate how many plants had a height that was greater than 45cm.

Box Plots

1) The ages of 20 teachers are listed below.

22, 22, 24, 25, 27, 27, 28, 29, 29, 29, 34, 35, 41, 43, 44, 49, 55, 57, 58, 58

a) On the grid below, draw a boxplot to show the information about the teachers.



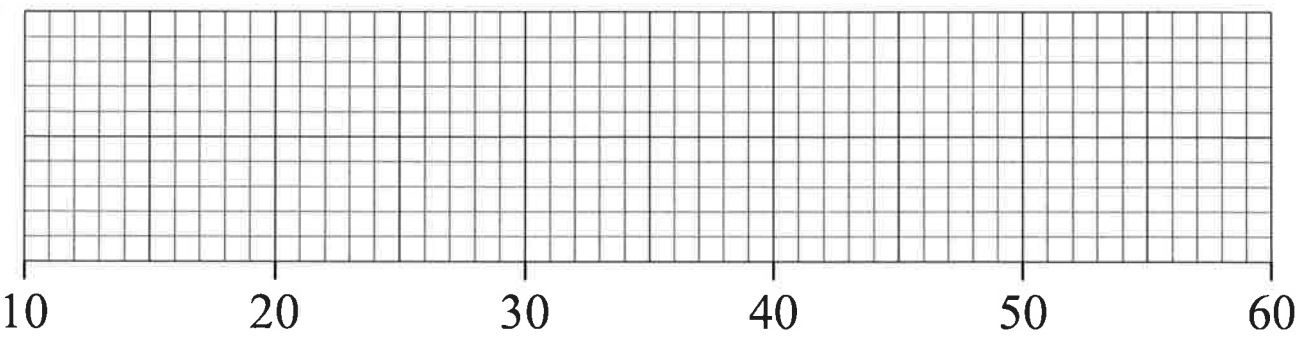
b) What is the interquartile range of the ages of the teachers?

2) A warehouse has 60 employees working in it.

The age of the youngest employee is 16 years.
The age of the oldest employee is 55 years.

The median age is 37 years.
The lower quartile age is 29 years.
The upper quartile age is 43 years.

On the grid below, draw a boxplot to show information about the ages of the employees.

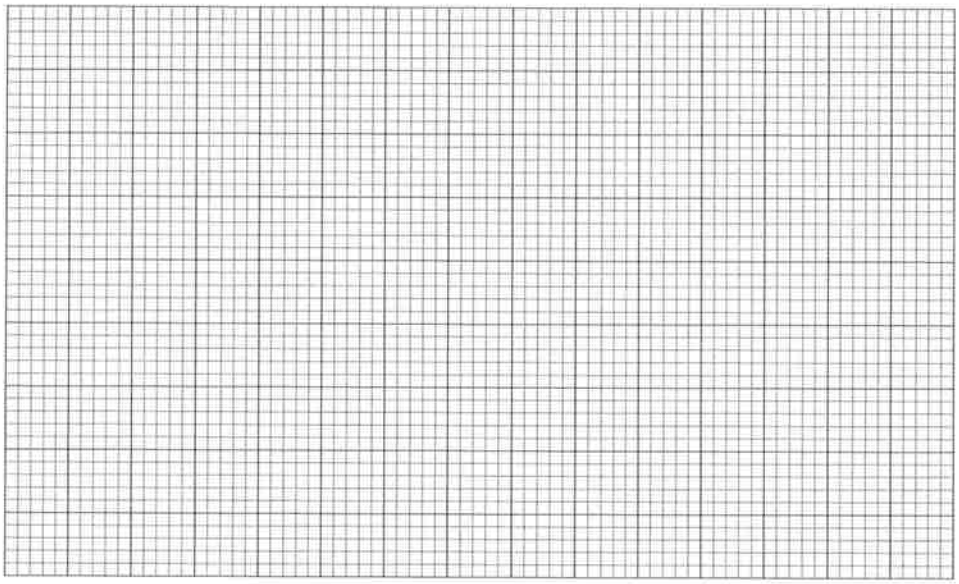


Histograms

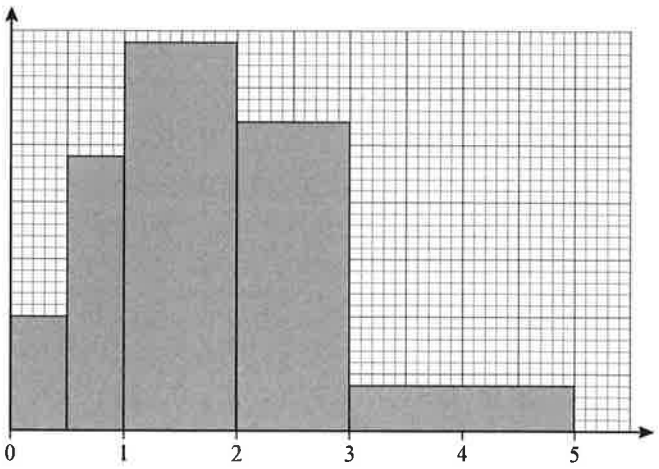
1. The table gives information about the heights, in centimetres, of some 18 year old students.

Height (h cm)	Frequency
$135 < h \leqslant 145$	12
$145 < h \leqslant 165$	46
$165 < h \leqslant 180$	45
$180 < h \leqslant 190$	25
$190 < h \leqslant 195$	4

Use the table to draw a histogram.



2. The histogram shows the amount of time, in hours, that students spend on their homework per week.



Use the histogram to complete the table.

Time (t hours)	Frequency
$0 < t \leqslant ½$	
$½ < t \leqslant 1$	
$1 < t \leqslant 2$	
$2 < t \leqslant 3$	27
$3 < t \leqslant 5$	

Tree Diagrams

1) A bag contains 7 green and 3 yellow balls.
A ball is taken from the bag at random and **replaced**.
Another ball is taken from the bag at random.

a) Draw a tree diagram to show all the possibilities.
b) What is the probability that both balls are different colours?

2) A bag contains 7 green and 3 yellow balls.
A ball is taken from the bag at random and **not replaced**.
Another ball is taken from the bag at random.

a) Draw a tree diagram to show all the possibilities.
b) What is the probability that both balls are different colours?

3) A box contains 5 red counters and 3 blue counters.
A counter is taken from the box at random and **not replaced**.
Another counter is taken at random.

a) Draw a tree diagram to show all the possibilities.
b) What is the probability of choosing at least one blue counter?
c) What is the probability of choosing two counters of the same colour?
d) What is the probability of choosing two counters of different colours?

4*) A box contains 4 red counters and 3 blue counters.
A counter is taken from the box at random and **not replaced**.
A second counter is taken from the box at random and **not replaced**.
A third counter is taken from the box.

a) Draw a tree diagram to show all the possibilities.
b) What is the probability that all three counters are the same colour?
c) What is the probability that exactly two of the counters are red?

And & Or Questions

1) Jordan designs a game for a school fair.
He has two 8-sided spinners.
The spinners are equally likely to land on each of their sides.

One spinner has 3 blue sides, 2 yellow sides and 3 white sides.
The other spinner has 2 blue sides, 2 green sides and 4 white sides.

Calculate the probability that the two spinners will land on the same colour.

2) The probability that it will snow in Paris on Christmas day is 0.06.

a) Work out the probability that it will snow in Paris on **both** Christmas day 2008 **and** Christmas day 2009.

b) Work out the probability that it will snow in Paris on **either** Christmas Day 2008 **or** Christmas Day 2009, but **not** on both.

3) A bag contains 2 black beads, 5 yellow beads and 3 red beads.
Natalie takes a bead at random from the bag, records its colour and replaces it.
She does this two more times.

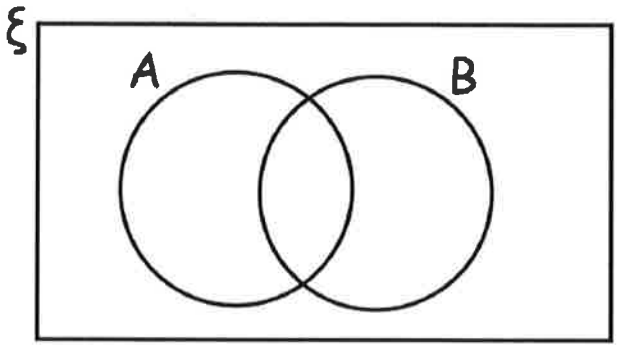
Work out the probability that, of the three beads Natalie takes, exactly two are the same colour.

Venn diagrams

1. $\xi = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16\}$

A = multiples of 3
B = multiples of 5

(a) Complete the Venn diagram



(3)

One of the numbers is selected at random.

(b) Write down $P(A \cap B)$

.....

(1)

2. In a company there are 110 workers.

90 workers like tea.
41 workers like coffee.
25 workers like both tea and coffee.

Work out how many workers like neither tea or coffee.

3. A group of friends have been surveyed.

38% have been to Canada.
80% have been to France.
11% have been to neither Canada or France.

Find the percentage of the group that have been to Canada and France.